

# Motion and Optical Flow

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Monday 1 Nov 2006

# video as spacetime block

- Set notation

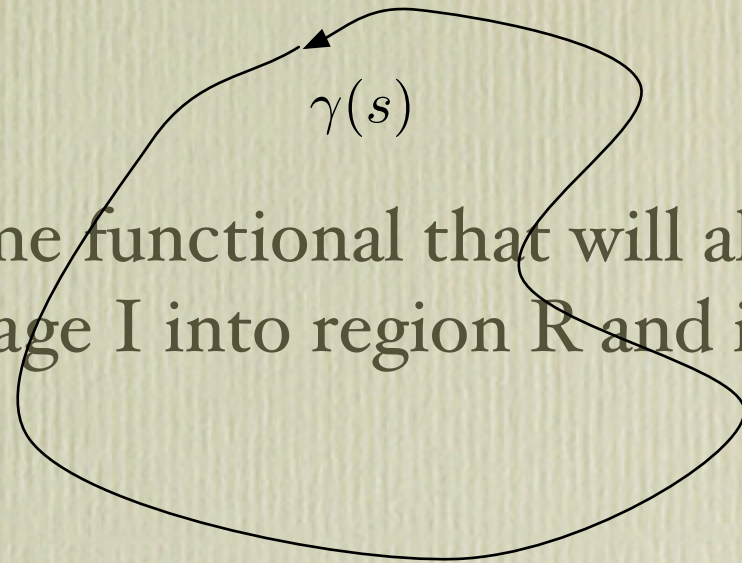
$\Omega$  is a rectangle in  $R^2$

$$I : \Omega \times [0, \infty) \rightarrow R^+$$

vector field  $x \in \Omega$

$$v(x) = \langle v_1(x), v_2(x) \rangle$$

# variational formulation: active contours



- We wish to define some functional that will allow us to partition the image  $I$  into region  $R$  and its complement  $I - R$ .

Let  $f(\cdot)$  be a monotone decreasing function, then we seek:

$$\min_{\gamma} \int_{\mathcal{R}_{\gamma}} f(\delta I(\mathbf{x})) d\mathbf{x} + \lambda \int_{\gamma} ds$$

Q. What does minimizing this functional favor?

# variational approach ...

$$\min_{\gamma} \int_{\mathcal{R}_{\gamma}} f(\delta \mathbf{I}(\mathbf{x})) d\mathbf{x} + \lambda \int_{\gamma} ds$$

- minimizing favors regions of large gradient of I, and at the same time controls (minimizes) length of boundary

Minimizing not over real numbers, but over function spaces: eg over curves,

Apply calculus of variations.

Solving Euler-Lagrange equations for that functional yields this differential equation, called an evolution equation:

# evolution equation

- Evolution equation for functional is an ODE to vary the boundary curve:

$$\frac{\partial \gamma}{\partial \tau} = F \vec{\nu} = (f(\delta \mathbf{I}(\mathbf{x})) + \lambda \kappa) \vec{\nu}$$

$\vec{\nu}$  inward normal to curve  $\gamma$

$\kappa$  geodesic curvature

As  $\delta \mathbf{I} \rightarrow \infty$ ,  $f(\delta \mathbf{I}) \rightarrow 0$

so the balloon force pushes contour to large gradient image areas

# sphere inversion problem

- old and new approaches

Thurston proof & video

Sullivan proof & video

using curvature-driven flow

# motion estimation

- different criteria for  
compression: motion-compensated  
compression (MPEG)

vs

motion-based video segmentation

skip many apparent motion effects due to  
variations in illumination or camera  
characteristics

focus on object-induced motion

# models of motion

- spatial models  
temporal models  
region of support

spatial model: assume that the movement of a dot at position  $x$  is modeled by some affine

map

$$v(x) = \begin{pmatrix} b1 \\ b2 \end{pmatrix} + \begin{pmatrix} b3 & b4 \\ b5 & b6 \end{pmatrix} x$$



# temporal model of motion

temporal model

assuming velocity is constant between time  $t$  and  $\tau > t$

$$\mathbf{x}(\tau) = \mathbf{x}(t) + v_t(\mathbf{x})(\tau - t) = \mathbf{x}(t) + \mathbf{d}_{t,\tau}(\mathbf{x})$$

- and ... region of support

# observation models

- Key assumption: Image intensity of a (point) object does not change along motion trajectory, so , for every  $\mathbf{x}$ :

$$I_k[\mathbf{n}] = I_{k-1}[\mathbf{n} - \mathbf{d}]$$

Differentiating w/r  $s$ ,  
where  $s$  is length  
along trajectory:

$$\frac{dI}{ds} = 0$$

by chain rule: 
$$\frac{dI}{dx} \nu_1 + \frac{dI}{dy} \nu_2 + \frac{dI}{dt} = (\nabla I) \cdot \boldsymbol{\nu} + \frac{dI}{dt} = 0$$

# regularization of image

- Underconstrained -- not enough conditions to yield a motion. Assume neighboring points move alike. One way: motion field is locally smooth, with low gradient. We minimize  $E[\mathbf{v}]$  for a velocity field

$$\int_D \left( \nabla I(x) \cdot \mathbf{v}(x) + \frac{\partial I(x)}{\partial t} \right)^2 + \lambda (\|\nabla(v_1(x))\|^2 + \|\nabla(v_2(x))\|^2)$$

# estimation criteria

- (Boldfaced are 2-vectors in  $\mathbb{Z}^2$  )  
 **$\mathbf{d}[\mathbf{n}]$**  = displaced image of point  **$\mathbf{n}$**  under the  
vector field  **$\mathbf{v}[\mathbf{n}] = \mathbf{d}[\mathbf{n}] - \mathbf{n}$**

estimated image intensity:  $\tilde{I}_k[\mathbf{n}]$

$$\tilde{I}_k[\mathbf{n}] \equiv I_{k-1}[\mathbf{n} - \mathbf{d}[\mathbf{n}]]$$

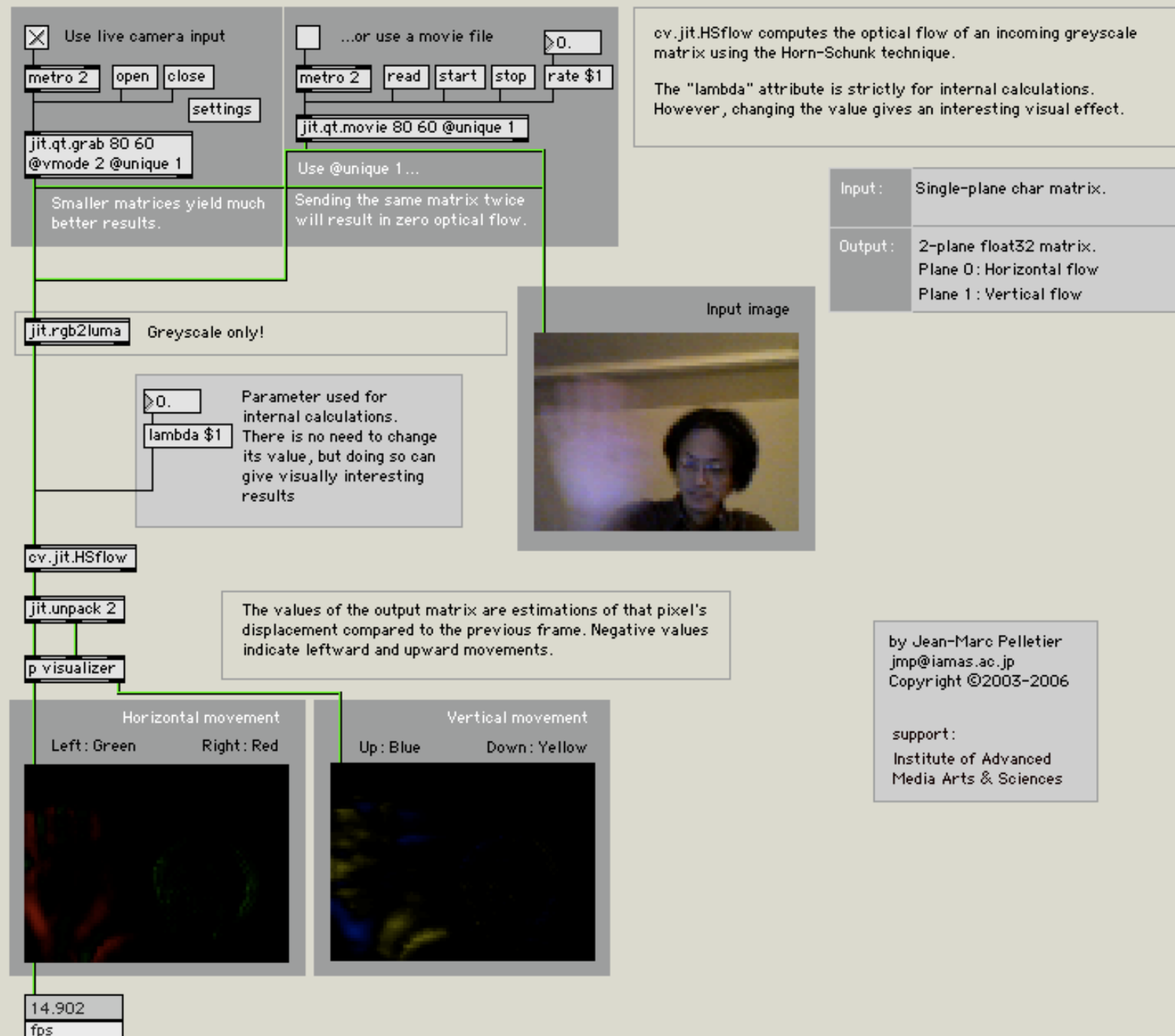
Find  **$\mathbf{d}$**  that minimizes an error function. A reasonable one is not quadratic (too many outliers) but simply:

$$\mathcal{E}[\mathbf{d}] = \sum_{\mathbf{n} \in \mathcal{R}} |I_k[\mathbf{n}] - \tilde{I}_k[\mathbf{n}]|$$

examples



## cv.jit.HSflow: Optical Flow

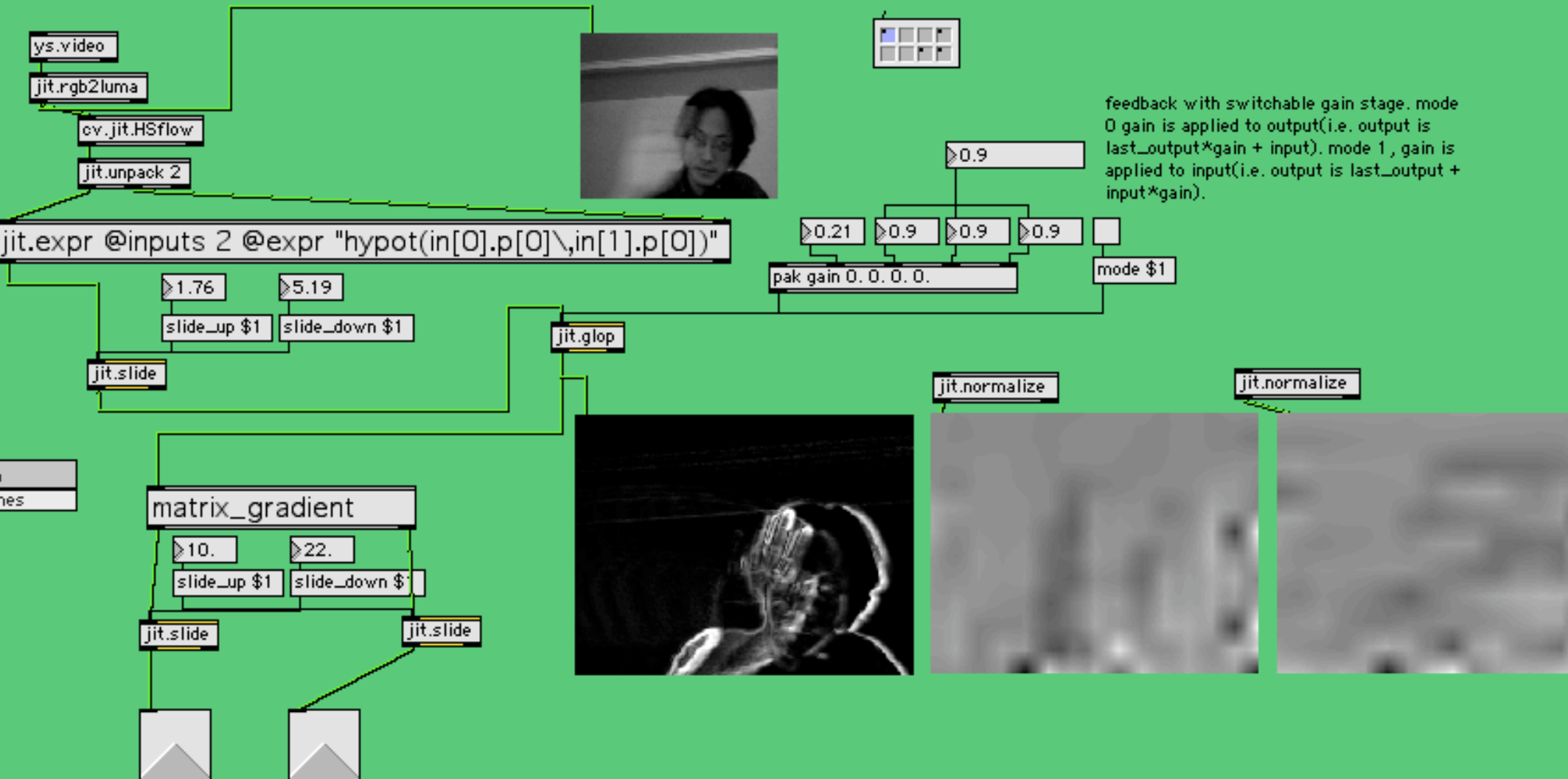


# matrix\_gradient\_blur

2\_matrix\_gradient\_blur

## matrix\_gradient\_blur

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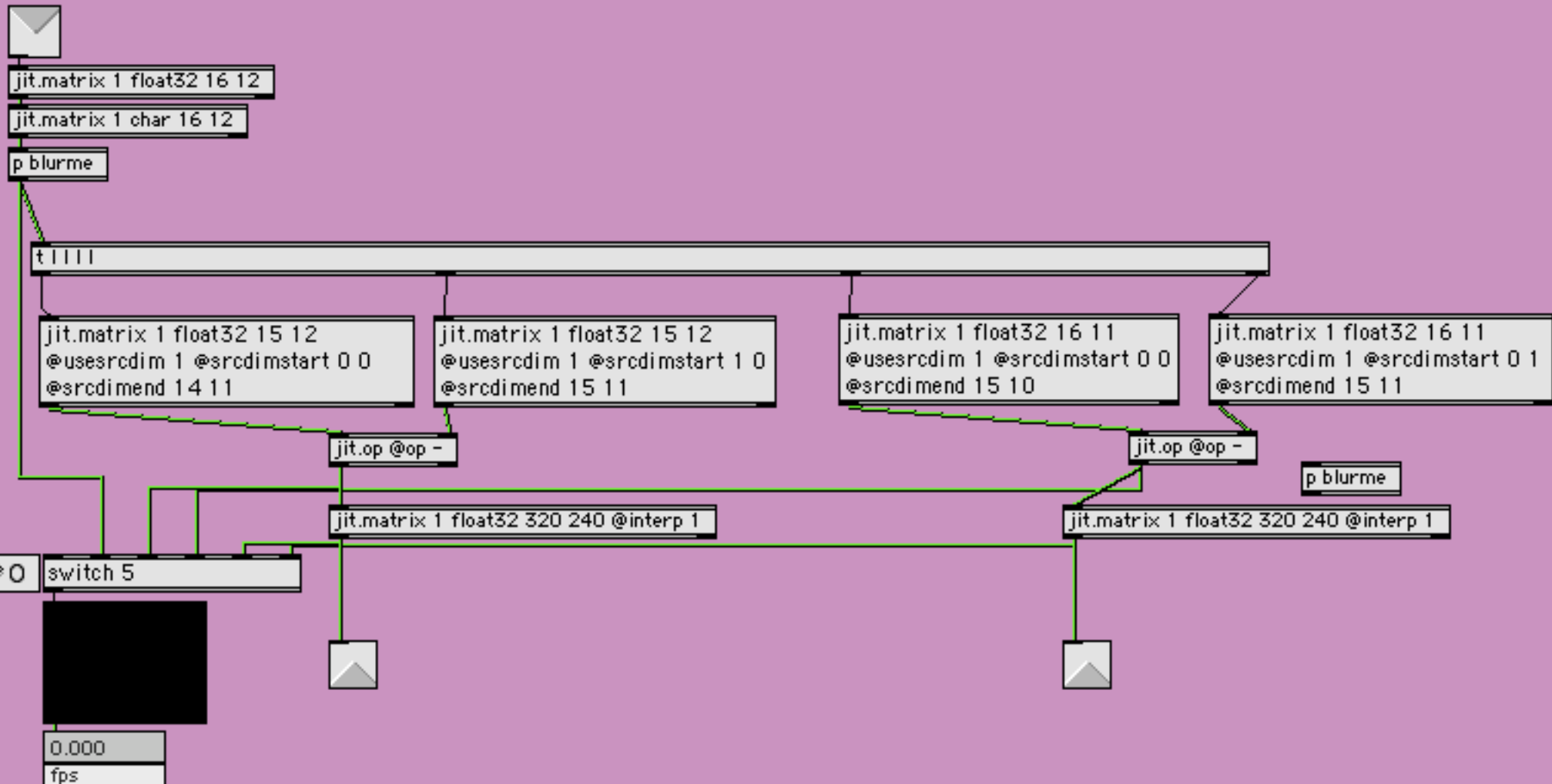


# matrix\_gradient

[matrix\_gradient]

## matrix\_gradient

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# Movie to Texture Grid 3\_test\_jit.gl.render.grid\_mesh

this patch shows creating a grid geometry, and texturing and displacing the geometry with a movie.

scale movie and map ARGBto RRGB

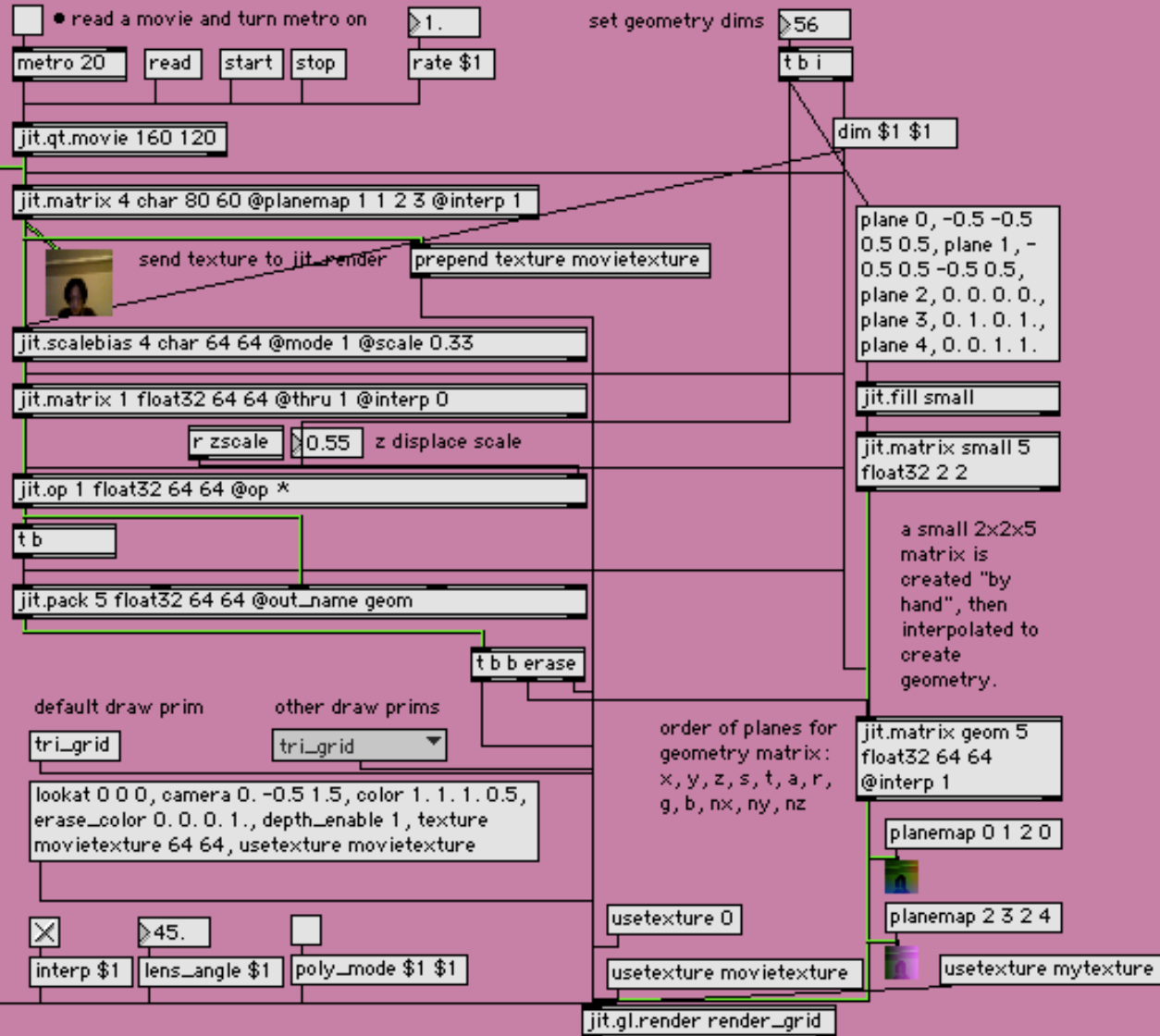
calculate sum of rgb components

convert to float32

multiply by a scalar

send to plane 5 (z) of geometry matrix

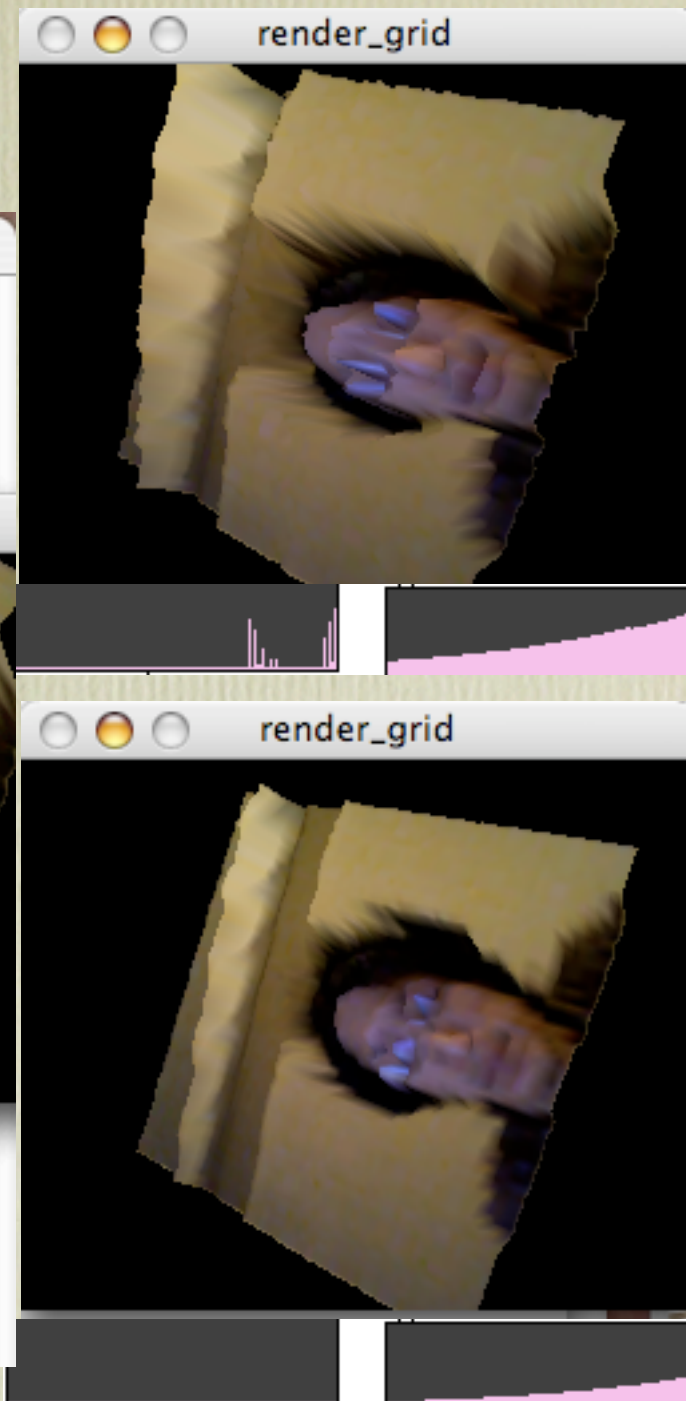
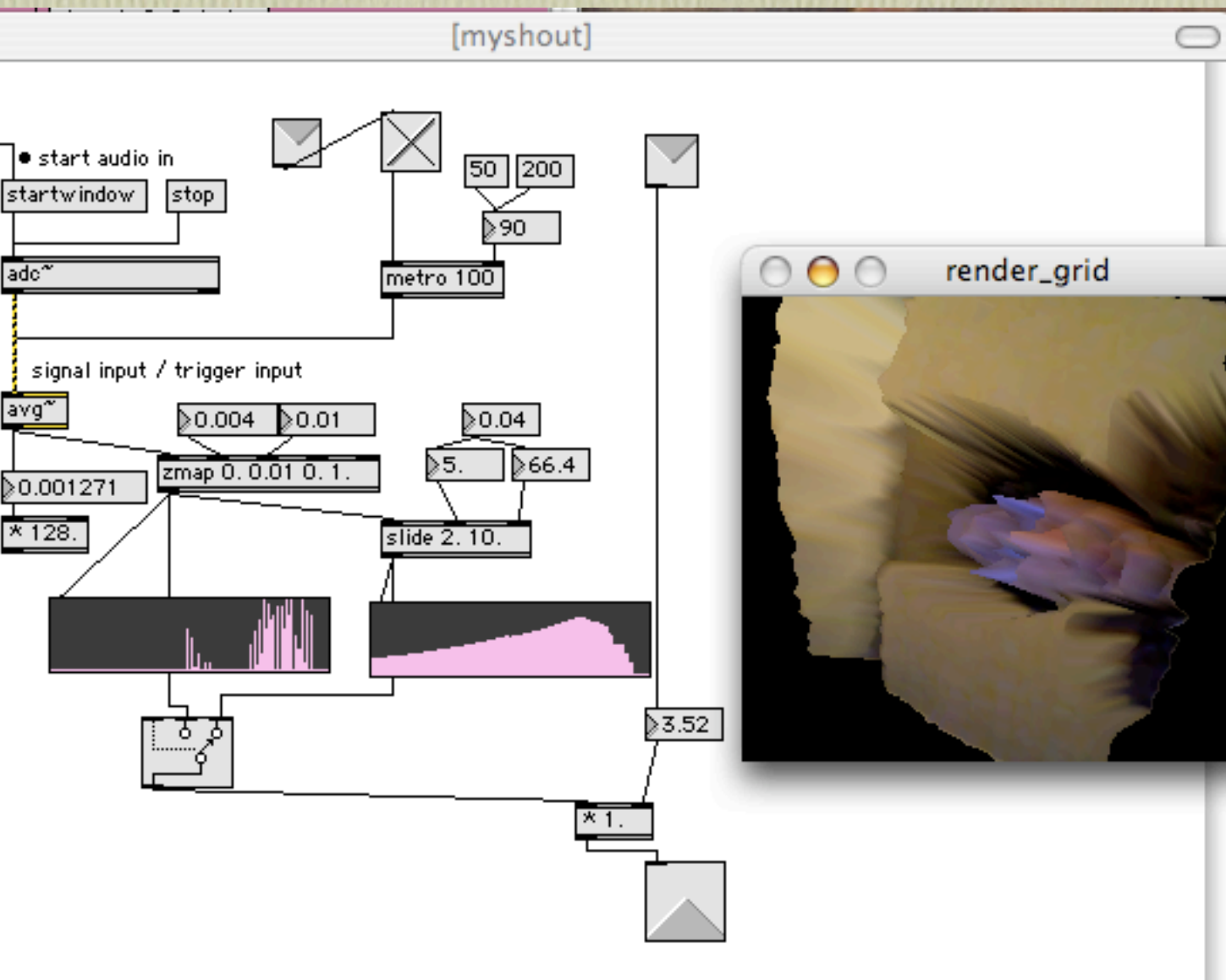
the window is made in here. drag in window to rotate the rendered geometry. hit 'esc' to toggle fullscreen mode.



modified by sha xin wei tml april 2006



# myshout

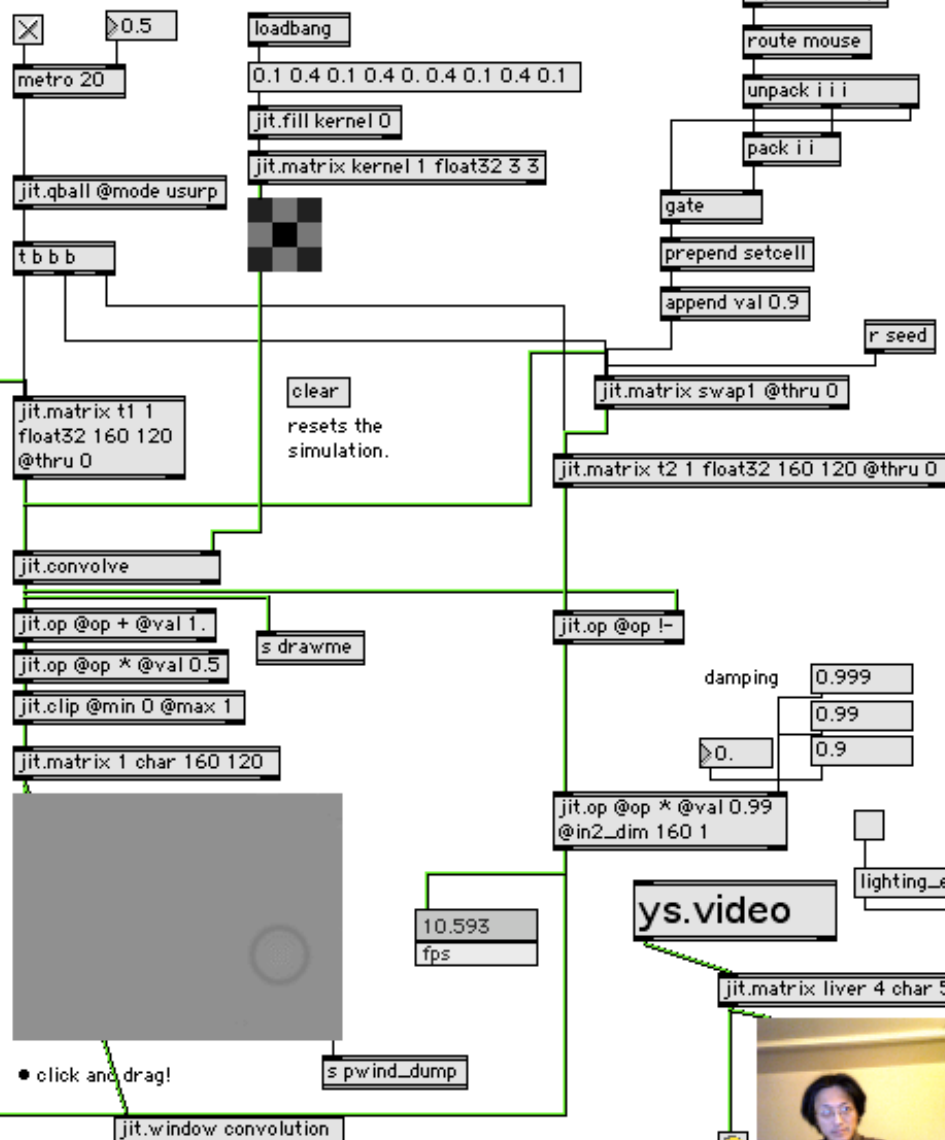


# pool 3d nurbs example modified sxw 10.06 live video texture

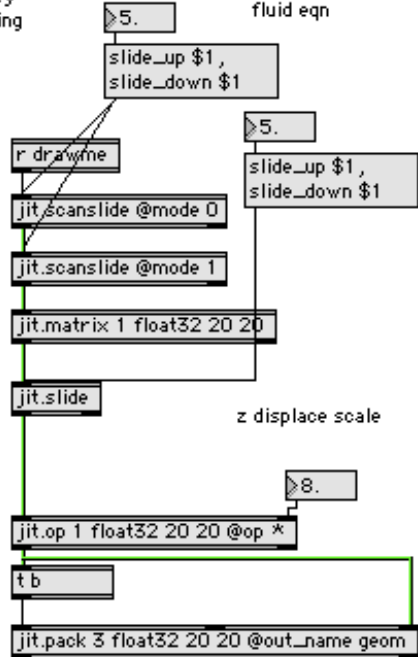
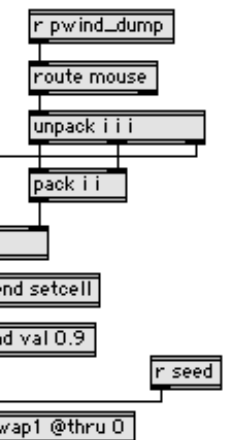
the surface motion of a contained fluid of constant volume is simulated using Jitter matrices and operators.  
 jit.convolve is used to add neighboring pixel values in the fluid equation. clicking on the pwindow sends a single 'splash'  
 to the simulation. try seeding the simulation using the "a" or "s" keys, or drawing in the window below. the "c" key  
 resets the simulation. try loading a texture and setting the tex\_map to 2 (sphere map environment mapping). setting  
 the nurbs dimensions to something larger like 80x80 produces smoother results, but runs slower.

use jit.scanslide to smooth the geometry, and jit.slide to soften the rapid oscillation determined by our fluid eqn

• turn me on!

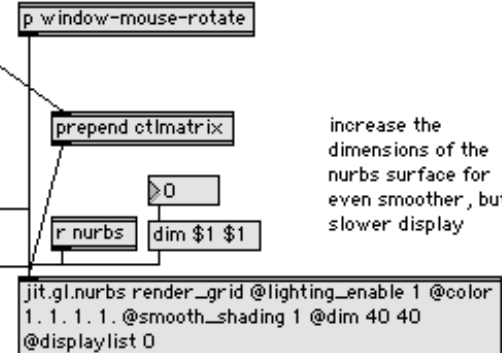
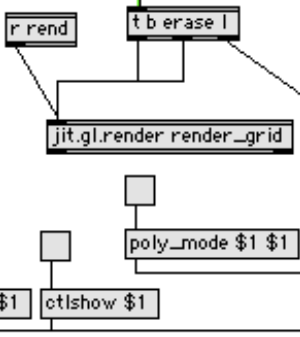
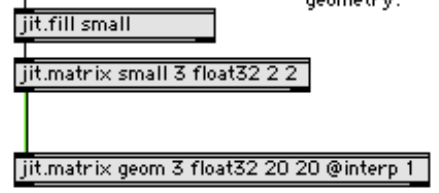


fluid eqn:  
 $(x, y, t+1) = \text{damping} * (1/2 * (h(x+1, y, t) + h(x-1, y, t) + h(x, y+1, t) + h(x, y-1, t)))$

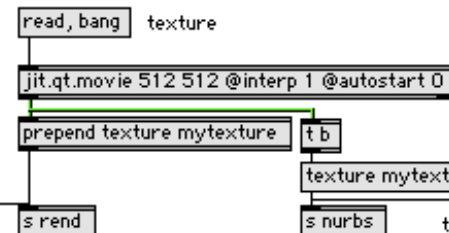
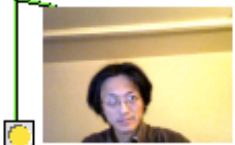


plane 0, -0.5 -0.5 0.5 0.5,  
 plane 1, -0.5 0.5 -0.5 0.5,  
 plane 2, 0. 0. 0. 0.

a small 2x2 matrix is created "by hand", then interpolated to create geometry.



increase the dimensions of the nurbs surface for even smoother, but slower display



try tex\_map 2 (sphere map environment mapping)

• click and drag!

10.593  
fps

# pool-3d-nurbs

