

The Medium of Contingency

An Inverse View of the Market

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The Vision Ahead

2.1 Regime-switching model

This book attempts to explain how any derivative pricing tool worthy of that name should work in the market. It attempts to understand what the market *is*, through the way in which the pricing tool works in the market. Recall that the pricing technology is both at one with the immemorial and immanent matter of the market and transcendent relative to the market – it is an a priori and apodictic stance – because of the element of design that is inherent in the technology. Recall that this point of view of the technology is what grants us access to the definition of the market. How a derivative pricing tool should work in the market is, first, by using probability theory and dynamic replication and, second, by recognizing that the matter that it is dealing with, the market of contingent claims, is precisely lodged outside any framing by probability theory or any fixed framework of states of the world. The regime-switching model will be the perfect illustration of this paradox when used as a derivative pricing technology and, accordingly, it will be our model for any derivative pricing technology.

Probability theory, Brownian motion, the BSM dynamic replication argument and the concept of implied volatility (which is how the BSM theory is applied to derivatives in the market and how it becomes a derivative pricing technology) form a historic and significant sequence which motivates writing this book. The reason why the regime-switching pricing model is the paradigmatic tool which summarizes and materializes this sequence (thus summing it up in what I call a technology, rather than theory) and why this book takes it up is that it precisely generalizes implied volatility. It builds precisely on the step outside probability theory and derivative valuation theory that implied volatility means, and it recognizes *matter* in that exit. More exactly, as we shall see, it gives its full material meaning to recalibration and recognizes the recalibration process as the true market process. So, it is not the philosophy that has imposed itself on the technology, but it is the opposite. It is the design, the purpose (and beyond that, the essence of the derivative pricing technology) that has changed the way that I think derivatives books (or more generally, books designed to illuminate the market from the point of view of the pricing of contingent claims) should be written.

Of course, the tool was not designed from scratch or by chance. The main insight is due to the designer of the tool – my partner in ITO 33, Philippe Henrotte – whose credo, which I share, is that derivative pricing and hedging as technology makes sense only to the extent that it is calibrated and recalibrated to the market of derivatives. *From prices to prices, with probability theory, stochastic processes and dynamic replication acting only as an intermediary step, or inner episode of the tool.*

It is sufficient to give it a minute of thought to realize that this new logic ('from prices to prices'), whose other name is recalibration, is incompatible with probability theory and the fixed and delimited states of the world that lie at its foundation. Ever since he graduated as a PhD in Finance and through his teaching years as professor of finance at HEC (École des Hautes Études Commerciales de Paris), the French business school, Philippe's main criticism against the Arrow-Debreu paradigm was that prices that come out as the result of probabilistic computations based on 'abstract' states of the world, according to Arrow-Debreu, will themselves constitute new states of the world (of the market). Hence a contradiction, or at least a perpetual breaching or non-closure of the ontological circle.

The regime-switching model has the unique feature that the regimes have no particular given names (of variables that are specifically stochastic – such as stochastic volatility, stochastic jumps, hazard rates, and so on) but adopt only the names that calibration assigns to them. Every option pricing specialist agrees today that models of the underlying dynamics must include stochastic volatility and jumps in the underlying price. Stochastic volatility and correlation between the underlying price and volatility are needed in order to explain deviations from BSM in longer-dated options, whose value is precisely sensitive to changes in volatility; and jumps are needed in order to explain deviations from BSM in shorter-dated out-of-the-money options, whose value, when they are so close to their expiration, no longer quite depends on volatility but is very sensitive to jumps instead. Indeed, the value of the deep-out-of-the-money option can vary massively depending on whether a jump in the underlying price can trigger it unexpectedly, when it was supposed to expire worthless. These deviations of options prices from BSM are known as the *smile problem* in the industry. Usually, the parameters of the stochastic volatility/jump-diffusion model are calibrated from the instant options prices, or the option volatility surface, or again the smile.

The regime-switching model is the recognition that the parameters will stochastically change, every time they are recalibrated. Hence, the 'real' model should be a collection of stochastic volatility/jump-diffusion models of different structure, each corresponding to a different regime. A particular regime is not characterized by a particular value of volatility or a particular size or frequency of jumps, but by a whole combination of such values. Moreover, two different regimes of volatility and jumps can be merged together and redefined as a single regime, thus preparing for a switching between two such superstructures. As a consequence, it will always be indistinguishable whether a given instance of the regime-switching model is a case of calibration or of recalibration (see Appendix A).

Now, let us try and think that the given sequence of recalibration is only an 'accident' of time (because we all happen accidentally to live in time) and doesn't reflect the real essence of the market, which is that no contingent claim, no matter how complex, should be redundant. Let us speculate, in other words, that the real process doesn't take place in time but on the 'spot' (what I like to call 'in

place'), and then the structure of the market, or its real process, will have no other place to go but into a dimension that has to be real although it is not actual – because actualization is what gives the actual prices we calibrate against in time.

This combination of reality and non-actuality is the stuff the future event is made of and the new matter that the book wishes to introduce. As I said, this is a metaphysical discovery or synthesis, so I guess it can have no causal agency of any sort and cannot change anything, except the way we think. It certainly cannot change the market, the tool, or the world, for that matter. What definitely changes, after the book, is the schema of stable, delimited and identified states of the world (i.e. the basis of probabilistic thought, and perhaps also of objectivist thinking in general) which should be replaced, I speculate, by the medium of the market and prices as translators of contingent claims.

The usual criticism of BSM is that it assumes Brownian motion with constant volatility. Constant volatility is unrealistic, of course, but what's even worse is that the market is complete as a consequence and derivatives are redundant. Assuming stochastic volatility and jumps makes the market incomplete all right and, as a consequence, derivatives are no longer perfectly replicable by the underlying; however, in reality they should no longer trade and admit of market prices for, if they did, their price processes could be used to complete the market. Since the whole pricing technology following BSM has the trading of derivatives as its purpose, I prefer as an alternative and less hypocritical characterization of the market in that register that *all derivatives should trade but that none should be redundant*. The only way to make that work is to forgo the structure of states of the world.

The book is a reflection on possibility, probability, state, time, and so on as metaphysical categories, and I propose replacing them with contingent claims, prices, money, and market dynamics as metaphysical categories of their own. You can see how far I place myself from the sociologists, behaviorists, econometricians, or indeed financial engineers and analysts of any following. I just deal with thought as my primary matter.

2.2 Recalibration

As recalibration is the name of the game, the book aims to give it a firm theoretical foundation. Recalibration is what all derivatives traders do when using derivative pricing tools and is precisely what derivative valuation theory, or academia at large, cannot account for. Derivative valuation theory deals only with stochastic processes and stochastic control and knows nothing of the concept of market price, even less so of the concept of implied volatility. The whole book can even be described as just an attempt to make sense of the BSM implied volatility (which is the simplest instance of calibration and recalibration). It is in line with the project I first started in *The Blank Swan*, as the following quote from *The Blank Swan* bears witness:

The concept of *implied volatility*, which is perhaps the most popular concept among derivative traders, is indeed the whole reason why I am trying to think this inversion in the widest scope that I can, and why the whole purpose of the book might be summarized in trying to find what

the derivative pricing theory, or at least the derivative pricing technology, could be, *given* the derivatives market, and not to try to build, like in all existing books, a derivative valuation theory, given the theoretical assumption of a stochastic process (and no derivatives market).¹

What the expression ‘given the derivatives market’ means is that a pricing tool is only as good as its calibration to existing derivatives prices and is only as useful as the subsequent trading *in the market* of the prices it outputs. *From prices to prices*, as my motto goes, *with theoretical valuation serving only as an inner episode of the tool*. This supplementation of valuation theory with a market that flanks it can only make sense as a *technology*. It is no longer a theory.

So, to be correct, we shouldn’t be speaking of ‘giving a firm theoretical foundation’ to the practice of recalibration but perhaps only a meta-theoretical or philosophical account – unless, that is, we made the market itself an integral part of this extended ‘pricing theory’ and no longer considered it as the exception or the accident that always ruins the probabilistic model! How to bypass probability theory and go directly from contingent claims (which are the only stuff the market is made of) to their prices (which should be the translation of the contingent claims) is where the book starts becoming ambitious, metaphysically (see Part III).

Now, supposing the metaphysical doctrine is granted, according to which contingency has to be mediated absolutely, without bringing in the relative frame of reference of possibilities and profiting only from the new matter that fills the void of possibilities, or that is overlaid above the successive failures of each and every schema of possibility; supposing, in other words, that the market is the place outside of time and time series that future contingent events inhabit and that trading in this market (via recalibration) is better than prediction – for prediction can only evoke possibilities and, therefore, come second, or maybe prediction should not even exist – we wonder how we could put this metaphysical doctrine to usage; how we, human beings who cannot escape the time-bound world, could experience this advantage of the market over prediction. Or maybe we never will and the answer will simply be that recalibration is all that we will ever be able to do when we are hit by contingent events.

Indeed, metaphysics doesn’t have physical agency and can only be put to usage in thought. However, I could have summarized my whole metaphysical elaboration in a single, down-to-earth and almost nonmetaphysical observation. Although they may lose money on the particular day that the pure contingent event hits the market (depending on their position), real market-makers make money afterwards just by virtue of widening their bid-and-ask spreads and the explosion of trading volumes. Everybody thinks this is a quite trivial matter. However, if we think about it for a while without any preconception, we realize that this phenomenon involves all the mysterious metaphysical categories we have been considering, namely, temporal succession, money, price, trading, the event, and so on and that by arranging them differently in thought, in a new inventive manner, we might end up with quite an original interpretation.

For instance, nobody said the market-marker’s profit and loss (P&L) should be accounted for chronologically, or at least day to day, and everybody agrees that successfully predicting an event in the market can only mean that you have made real money out of your prediction (nobody cares about

your private knowledge or private intuition). In other words, reasoning outside time and replacing a mysterious concept (prediction) with its only tangible and clear consequence as far as consistent work in the market is concerned (making money), we can safely declare that the market-maker has successfully predicted the event.

On the face of it, all this may sound like a mere substitution of words; however, it is not neutral, metaphysically. Indeed, what interests me is the reworking, from there, of the whole concept of the market as the medium of contingency and of money as the corresponding reworking of the concept of time.

Accordingly, the market shouldn't be thought of as a complex arena, filled with traders and information flows, in which informational efficiency or rationality can be measured and in which the conclusion is necessarily reached that the market will never settle in equilibrium but will always wander in chaotic dynamics. The market is not complex and shouldn't borrow any of the usual images of complex systems – the market as a living organism or as a neural network or as dynamic chaos. The market is simple. It is the place where contingent claims receive prices. To repeat, the book addresses a simple question: 'What is the meaning of implied volatility?' There is no knowledge in the market to begin with and there is no uncertainty. There is only trading. Implied volatility is a trading concept. My whole idea is that the regime-switching framework is the only answer to implied volatility or to recalibration. People have sought improper ways of generalizing BSM in front of the market. They thought they would generalize the underlying dynamics. Rather, it is implied volatility that needs to be generalized, not volatility.

Implied volatility means and implies stochastic implied volatility for the simple reason that the volatility you will imply next from the market option price will be different from the one you implied previously. In short, implied volatility is at once stochastic implied volatility – a paradox that challenges the ordinary logic of subject and predicate. Analysts wrongly thought that the volatility of the underlying should be made stochastic as a result; therefore they generalized the underlying dynamics of BSM to stochastic volatility models (Heston, and others).

In reality, what should be generalized and made stochastic is not volatility, but *implied volatility* or the fact that is implied by implied volatility; namely, that any parameter of any model of any complexity that we can think of will turn stochastic when it is implied. This is called the 'problem of recalibration'. This is the crucial problem of derivative pricing. It distinguishes derivative pricing technology from derivative valuation theory.

I claim that the regime-switching model is the (perhaps only) answer to the problem of recalibration, or to the problem of generalizing implied volatility (not volatility). The regime-switching model is more than just a model of the underlying dynamics. In any given fixed frame of reference, it relatively behaves like all other stochastic processes; however, the true (absolute) process that it reflects is the process of *non-redundant* differentiation of the successive derivative payoffs. This is not a temporal process. This is the real process of the market.

The *pricing process*, or the real process of the market which cannot be a temporal process, is such that no contingent claim should ever be redundant. In the market, we have price series, not time series.

Traditionally, the reaction has always been to make the underlying dynamics more complex in order to save the contingent claims from their redundant fate. The market is made incomplete by adding stochastic volatility or jumps to BSM. However, such a move doesn't recognize the scope of my criticism. I am not criticizing stochastic volatility or jump-diffusion, and so on, as such. These are very nice stochastic models, perfectly fit for derivative valuation. I am criticizing the insufficiency of *any* model of the underlying dynamics when confronted with the problem of recalibration, which is another word for derivative pricing. The real market dynamics is the dynamics of recalibration, not the dynamics of the underlying. There is no underlying and overlying in the market, no such hierarchy. All we have are contingent claims, all trading and admitting of prices at the same level.

2.3 Is probability necessary?

Recently, a famous economist spoke of a 25 percent probability of a double-dip in the economy by the end of the year. What does this 'probability' $p = 25\%$ mean? If I am not mistaken, because we are talking here of a single-case probability and not of one inferred from a statistical frequency, the metaphysical presupposition is that we are facing a certain number of possible worlds (or possible repetitions or rehearsals of history) $p\%$ of which contain the double-dip and $1 - p\%$ don't. Because the event is empirically a single case, we replace the thought of the statistical population in which the frequency of the event would have been empirically observed had it been repeatable and not single-case with the thought of an instantaneous 'statistical' population of alternative worlds, only a single one of which will be realized.

Contrary to the empirical population, these possible worlds are just an image in our heads whose sole purpose is privately to help us know what we mean when we talk of probability. Note that some authors, the so-called modal realists such as David Lewis, believe those possible worlds really exist metaphysically and are not mind dependent.² Others have suggested that those worlds neither really exist metaphysically nor are lodged in our heads; rather, they literally *are* the meaning of the word 'probability'; they are objective in the sense that meanings 'are not in the head' and are objective. This is the doctrine known as 'objective semantics'.³

In sum, all we have done is invent a word, 'probability', and invent more or less intricate ways of telling ourselves what we mean. De Finetti has proposed putting an end to the metaphysics of probability (his famous: 'Probability doesn't exist!') and has identified probability with the quote some *banker* is prepared to give you for buying the corresponding contingent claim.⁴ In this case, the contingent claim would pay off \$1 at the end of the year in case of a double dip and 0 otherwise, and would be worth \$0.25 today. Those still inclined to use the word 'probability' have equated this 'transaction' with the fact that the banker *believes* that the metaphysical probability we have described above (the one involving the possible worlds) is 25 percent. Put succinctly, those worlds don't really exist (so probability doesn't really exist); they only exist as a belief.

In fact, de Finetti has just stopped one step short of admitting the market of contingent claims, and of replacing all talk of probability with talk of price! We all know that a *single* banker doesn't exist. Only a crowd does. What stops all the other bankers from entering the game and auctioning

this contingent claim; that is, turning the quote into a price? This is our credo: From the moment the thought of the metaphysical possible worlds (or possible states of the world) is transformed into a *written contingent claim* which pays off in case of the event – in this lies, to our mind, de Finetti's crucial step – the market, the crowd, and hence the price are created. If you invite one banker, you invite them all. In reality, de Finetti was still attracted by the image of the possible worlds. However, because he very strongly felt those worlds shouldn't really exist, he had no choice but to interpose a *believer* in them.

Objection: But then, surely, the market consensus (i.e. the price) is some aggregate of individual beliefs? Answer: What do you mean by 'aggregate of beliefs'? Is this aggregate still a belief? If not, what is it? Using the word 'aggregate' immediately prompts the dynamic image of different beliefs entering into conflict in the market arena, and finally settling on the consensus. But why even bother with this image? I say forget the individuals and forget the beliefs. We know what the price of the double-dip contingent claim is at the end of the year. It is \$1 in case of double-dip, 0 otherwise. Let us then *define* the market as the medium in which contingent claims admit of prices *ahead of their maturity*. Better, let us define money as the piece of paper one unit of which you exchange against the contingent claim at maturity in case of a double dip and 0.25 unit of which you exchange against it ahead of maturity.

How can such a thing as money exist? We create it. How does the contingent claim exist? We write it (using the previously created notion of money in its written contract). How does the exchange place exist? As we have said, we just define it as the place where prices attach to the contingent claims. Why would the 'place' where probability is attached to states of the world be more legitimate and more readily available? Probability is a confused and conflicting concept; it is unsettled (literally lacking a proper dwelling place), not knowing whether it should attach 'objectively' to the event or only 'subjectively' to the mind of the believer. I still haven't seen the place where probability attaches to things or to minds. By contrast, I have seen the marketplace.

Moreover, there is homogeneity in the medium of prices, which is absent in the medium of probability. Both the \$1 price and the \$0.25 price are prices; the contingent claim is exchanged against money in both situations. By contrast, a probability of 1 and a probability of 0.25 point to different categories. The former characterizes reality and actuality; it is the realization of the event. The latter characterizes only a possibility. (Our whole metaphysical problem is to understand the relation, or the difference, between the possible and the real.)

I think the only reason why we have imagined states of the world and why we identify them clearly is to stage the *transition* between the possible and the real. I think the raw fact (what I call the 'massive and unsophisticated real') is that the world is contingent, which means that it is not necessary, which means that it could have been different. The unwarranted step that metaphysics then undertakes is to *identify* the different ways that the world could be, by arbitrarily isolating the so-called states of affairs or states of the world and varying them, and to make those alternative worlds coexist with the present one in the only place where such coexistence wouldn't violate the principle of excluded middle. This place is located one step back, at the imaginary moment when neither the present world nor its alternatives are supposed to be realized. To repeat, what is real is

the one and only present world and *what is also real* is its contingency. However, metaphysics has no other way of representing this than by stepping back into unreality. It stages the possible then it has to stage its realization.

Every day the world is real, and every day it could have been different. Metaphysics has no other way of chaining together these differences in time than through a tree of possibilities and probabilistic transitions. The contingency of the current world is referred back to the previous step when the world is just one among a certain number of possible worlds. Only one world is realized at a time and no two worlds can share the '1' of reality at a given time; so the difference is referred to numbers that are less than 1 (probabilities), which the different worlds can share at the previous step.

Now, what if we tried to make the picture more robust and no longer identified the possible variations of the present, massive, contingent world? What if we applied Occam's razor and admitted only that the present world *can be* different without explicating (enumerating) what other worlds it may *be*. In other words, instead of thinking that the verb *can be* is derivative on the verb *to be*, why don't we think the opposite? (Why can't contingency become the basic ontology?) How can we think of the succession between the one and only real (yet massively contingent) world and the one and only real (yet massively contingent) world that succeeds the previous one without mediating the succession by a transition between identifiable states?

When we write a contingent claim that pays off \$1 next year in case of a double dip, 0 otherwise, and a contingent claim that pays off \$1 next year in case of no double dip, 0 otherwise, we don't think of two different possible worlds. The two contingent claims coexist today in the same real world; and so will they tomorrow and next year. They both have '\$1' written on them, whereas two different possible worlds cannot have 'probability of 1' written on them simultaneously. The two contingent claims will certainly admit, today, of prices that are less than \$1. Otherwise, there would be arbitrage. On the other hand, nothing imposes that the prices must sum up to \$1 today (assuming zero interest rates). For all we know, nuclear war might break out next year (or any other contingency, such as all trades being cancelled, and so on), and the market might be 'discounting' this already. Or maybe their prices will sum up to \$1 today, and the arbitrageur who has bought the bundle of them will lose money in case of nuclear war. This is not a problem, as there is nothing wrong with losing money in the market. By contrast, if your range of previously identified states of the world shifts unexpectedly, probably you will lose money too; however, there will have been something wrong with the probability view.

I think the duality between (K, T) (strike and maturity of the contingent claim) and (S, t) (state of the world, time) – or, in other words, the duality between the writing of contingent claims (and ipso facto their market) and the fiction of possible states – was not available to the probabilistic thinkers when they thought of their possible worlds and devised their metaphysics. Probability is backwards, whereas price is forwards. If you value options in the (S, t) picture, you have to work backwards. You fix the future states of the world (under what authority or right?) and you roll backwards to price a single option. By contrast, you work forwards in the (K, T) picture. However, you don't move forwards in time or to another possible world. You price all options of different strikes and maturities (K, T) standing in the spot $(S = S_0, t = t_0)$.

So, the real world containing the contingent claims and their prices succeeds to the real world containing the same contingent claims and different prices. Or rather, it is the same world. There are no transitions, only price dynamics; that is to say, constant recalibration to the market of valuation models that internally respect the no arbitrage principle. But how is the market functioning? How is the price emerging? What is *the* model of the market? I am sure one can endlessly think of the inner workings of the market. However, what interests me, at the higher level, is the short-cut. What interests me is that the picture (contingent claim, price) should be more robust than the picture (state of the world probability). As I said, there is no need to identify states of the world underlying the prices, so long as the market ‘massively’ (i.e. indistinctly) takes care of the transition.

In this sequence of thought, I am not trying to explain what the market is. Rather, my derivation (and my motivation) can be reconstructed as follows. I want to express the contingency of the world; however, I want to do that while getting rid of possibility and the corresponding metaphysics. I then observe that printing money and writing contingent claims is the best way of ‘flattening’ the depth of possibility and the corresponding tree; that is, the best way of materializing the possible worlds and collapsing them into the only real one (at work, here, is a kind of magic that is called ‘writing’). In a last step, I look for the medium that can connect the present day with the maturity of the contingent claim. I call this the market.

We should try and deduce the market and the exchange place directly from the contingent claim, without even assuming the existence of market participants and of exchanging parties. Simply, while possibility needs *chronological time* in order to step back and forth, from the fictive alternative worlds to the realization of the real world, contingency needs *place*. Possibility is a sort of bond, or debt. We alienate the real from its reality in order to fabricate possibility and we are bound to redeem its reality by realizing the possibility. By contrast, the contingent claim is the *conversion of debt*.⁵ While debt is tied up to the debtor who can alone redeem it (unless debt is exchanged on the market, of course; but the market doesn’t exist yet; we are precisely in the process of creating it), the contingent claim is tied up to nobody in particular. Instead of a convergence to face value, it finds divergence and the open place where anybody *can be*. Hence the marketplace.

2.4 Price and probability

Price is not a probability or, equivalently, a mathematical expectation (as orthodox financial theory has it). As my philosophy is directed against the foundation set by Arrow-Debreu – that is, against the notion of basic states of the world and against the idea that basic securities that pay off \$1 in those states of the world should be the building blocks of the pricing theory – I reject the notion that the price of these things is the discounted (risk-neutral) probability of the corresponding state of the world.

Not that such securities shouldn’t trade when their payoff condition is well-defined (I mean when the corresponding state of the world is something tangible and not some *deus-ex-machina* ‘state of the economy’). But I need their price to be given by the market, not by probability.

And what is the basic, to this day unanswered, criticism of Arrow-Debreu? What is, in a nutshell, my basic argument against the identification of states? Simply, that the prices of those Arrow-Debreu securities, if and once you compute them by probability, become additional states of the world/market, because the purpose of the whole exercise is to *trade* them, not to value them.

In a nutshell, my philosophy tries to make sense of the fact which orthodox valuation theory cannot account for – the fact that the derivatives we are valuing have to be traded, therefore, have to diverge from their theoretical value. Herein, to my mind, lies the scandal of valuation theory (to be rejected) and the paradox of the pricing technology (to be philosophically accounted for).

My book aims to make sense of the process of *recalibration* of the pricing tool. Derivatives traders use probabilistic valuation models that are supposed to describe the dynamics of the underlying and its statistical properties once and for all, or at least until the expiry date of the derivative; yet, they *recalibrate* their model every day against the market prices of derivatives, thus inverting its usage and perverting its foundation. Indeed, the assumption of statistical regularity, which is the very basis of any probabilistic representation, is contradicted every day following such recalibration and the parameters of the model are changed every day. How can probability theory and identification of states stand in the market when, by pricing, trading and recalibrating every day, you are going to defeat the model every day, expand the states of the world, and ruin the probability calculation?

I discovered (yes, a true discovery) that the philosophy of the event and of contingency can alone account for this constant context-changing feature of the recalibration process. A superior representation of contingency and of time is needed, the one found in Deleuze's philosophy (and his interpretation of the Nietzschean dice-throw).⁶

Nobody should stop using probability as a tool. Arrow-Debreu prices are given by the market; this is why they are called 'prices'. And the only probability that should exist, to my mind, is what financial theory calls *risk-neutral probability*; that is, the one that precisely helps us interpolate derivatives prices from existing (i.e. trading on the spot) derivatives prices, pending the recalibration that will ruin the structure of states of the world underlying the passing probability computation.

What I reject is the metaphysical view and schema according to which probability is assigned to 'real' and subsistent (in the metaphysical sense) states of the world. To repeat, I believe that the metaphysical presupposition of econometrics and of orthodox financial theory – namely, the existence of a data generating process (DGP) underlying the market – is not true. Only instant prices (of derivatives and underlying alike) exist. That you should temporarily posit states of the world in order to express the risk-neutral probability is what I call 'using probability as a tool'. However, this is only for convenience. Philosophers call this view 'instrumentalism' or the belief that theories are not real, but mere instruments.

2.5 A new metaphysics

The pricing model that I defend in the book, the *regime-switching model*, is a probabilistic model like any other; therefore, any instance of it relies on states of the world. But the real killer is recalibration. The problem of recalibration is what drives me, *metaphysically*, to search for a

substitute to probability. As I said earlier, my book is a book of metaphysics. And the reason why I discuss the regime-switching model is that it sustains recalibration, or so I argue.

Metaphysics won't change the trader's trading habits, I know. However, I believe that it is, in the present instance and problem, the only way to answer the question: 'What is the market and what is price?'

My main insight is the following: Instead of thinking that the market is a complex exchange phenomenon (human, behavioral, what have you) that will never be tractably modeled by probability theory, why not take advantage of the necessity to recalibrate the derivative pricing tools in the market and *define* the market as this alternative medium where the process of recalibration (which exterminates probability) is the rule? Why not postulate that the market is a *simple*, not a composite. That it is the *medium of contingent claims*, and that it is such that contingent claims are never redundant?

In my metaphysics, I need the market, or the price (of any derivative), to be an entity that is always rebellious to a partitioning into states, and ultimately rebellious to redundancy. I do not define the price through the exchange. I do not consider the market as made up by human beings. My work is speculative philosophy. I am building a system of metaphysics.

In my view, the existence of contingent claims and of their market is a blessing for any thinker in literary theory who wishes to investigate writing as a process whose metaphysics is foreign to chronological time and even to biographical time. See the case of Pierre Menard.⁷ Indeed, the 'financial' entry point allows me to broach my topic (writing) from its sharpest angle, as it points my readers to a very material and pressing entity which directly affects their lives and which they will all agree does exist (the financial market), while not everyone agrees, by comparison, that writing (as such) is very material and that books (as such) materially exist and can change one's life.

Of course, everyone agrees that books empirically exist and that they can, on occasion, change the reader's life. What I mean to say is that the book, or generally the medium of writing (thus the market), exists in the sense that the *world* as a whole is said to exist, that the book (or the market) is in fact an *alternative* to the world and to its metaphysics altogether.

Indeed, my contention is that the market of contingent claims, when it is understood as a writing medium and not as a theatre of fixed possible states and probabilities (what philosophers call 'representation' precisely), evades the order of chronological time and its spatial correlate (the identifiable possible states) altogether. Metrical time and space are extensive dimensions that can only receive extensive variables. Price, by contrast, is an intensive variable.

From my first day as a trader on the floor, I was struck that something was going on, definitely not reducible to probability or to the metaphysical idea of a data generating process. Something singular, yet repeated. A constant Black Swan, so to speak. A constant revision and shift of the whole set of previously identifiable (if they ever were) states of the world.

The lesson I take from Taleb's book (*The Black Swan: The Impact of the Highly Improbable*) is that Black Swan events are shifts of the whole range of previously identifiable possible states. So, it is a misrepresentation to call them highly improbable, since probability is only defined relative to a fixed range of states of the world. Only quantum mechanics is able to offer a formalized predictive theory

that subsists at the meta-level where the ranges of possible states are not yet identified. The wave function is just a meta-probabilistic predictive tool. I am not saying that randomness in the market is quantum mechanical! My only interest is that such a criticism and surpassing of probability does exist, and is even provided by physics.

2.6 Absolute contingency

I am not the first to criticize the category of possibility. Neither is Taleb the first to invoke the backward narrative that is typical of Black Swans. To my knowledge, this criticism first came from Bergson, who is famous for saying that real events create their own causes. Deleuze and Badiou followed as philosophers of the event and contingency, and a recent brand of speculative philosophy (Quentin Meillassoux) even argues that, since contingent events escape the range and whole metric of possible states almost by definition, then there is no reason why we should not make contingency the absolute ‘matter’ – from which I deduce that we should define it positively and no longer only negatively relative to possibility.⁸ Why not, in fact, overturn the whole ontology and the whole order of thought and place contingency (the fact that a thing *can be* something or other or, as a matter of fact, that it *can not be* at all) before being (the fact that a thing *is*)? My whole point is then to ask what the medium of such absolute contingency may be, if it is no longer to be mediated by the category of probability.

My answer, as you may have guessed, is that the medium of absolute contingency (i.e. not derived from states) is the market of contingent claims, and generally the market. It is writing rather than prediction. Notice that we are here delving in the domain of speculative philosophy, which only deals with categories of thought a priori, such as the *real* or the *possible*, not with specific empirical domains or scientific theories. It is at this pure level of thought that I suggest we shift to the market of contingent claims as an alternative to possibility. Such is, indeed, my philosophical ambition for the market.

At this stage, it is no longer the ‘mundane’ derivatives market that I am talking about, the one made up by individual traders or psychologies or historical events. The category that allows me to abstract the market to such a level is, once again, writing. There is a deep insight from Derrida at play, here, to the effect that writing is not low and mundane; it is not a derivative activity; to the contrary, it ranks higher than being and is ‘older’ than being (as Derrida says⁹).

If price is to become such an original and *early* alternative to probability and written contingent claims are to become such an early alternative to possible states (early in the process of thought, that is), then even the phenomenon of the exchange has to be thought purely and metaphysically. More to the point, if indeed the market is a metaphysical answer to a speculative philosopher’s worry, then it should be possible to reconstruct the market purely from a priori philosophical arguments. (Meillassoux has even suggested that I no longer call ‘market’ this new category of thought, but ‘arche-market’.) This is what I achieve in the article ‘The Medium of Contingency’, where you will find the name of the market uttered only at the end.¹⁰

To repeat, the market, when elevated to the metaphysical ambition that it deserves, offers a real opportunity to thought which it truly belongs to the philosophers to understand and ultimately

assess. I agree that such a metaphysical speculation will not change the trader's trading habits and will even disturb financial engineers and quantitative analysts, or 'quants', who care only to solve 'quant' problems. But shouldn't we feel at least marginally concerned that the market should lend itself to such a philosophical overturning of thought? Aren't we in the least interested to see the two meanings of the word *speculation* (philosophical and financial) finally come together?

2.7 The market as an opportunity for speculative thought

Possibility is not real (following Bergson), while the market is. That the market should be real, that the prices of contingent claims should be real and non-redundant (and what could be the meaning of 'redundant' when no states of the world have been fixed?), all this points to the necessity of overturning the metaphysical presupposition of states of the world and of probabilistic transitions occurring between them and of looking for an alternative way of solving the problem of *succession* or of the passage of time.

As I have said, I firmly believe in prices being the only real. I just don't believe that you can freeze prices in a fixed representation of states of the world and call this a market. Not that such a representation exists only constantly changes. It simply doesn't exist. I simply define the market as this alternative ontology where the word 'state' no longer has meaning. This is the new metaphysics I propose. There are no mathematical proofs or empirical evidence of what I contend. This is philosophical speculation. I offer a new way of thinking the market, and consequently, of thinking the rest of the world. The problem of recalibration is real, of course; it is not metaphysical. It is just up to us to decide whether it is caused by the probabilities (and consequently to call this a 'defect' of probability) or by the prices (and consequently to call this a 'property' of price, perhaps even its 'essence').

Probability models shouldn't be discarded. They cannot account for the market and for the meaning of price, yet we need them in order to insert the market-maker in the dynamic replication process, and subsequently in the recalibration process. As we will see in the last part, the formalism is needed in order to grant the way out of the formalism systematically. This systematic overflow is what recalibration will amount to and it takes place after the contingent claims are irreversibly written and create, with their writing, a material barrier that blocks the return inside the formalism.

Probability is only an episode and a tool in the hand of the market-maker. I don't think we should look at the market from above, as if from nobody's point of view. Seen from above, the prices of derivatives and of the underlying alike just follow the movements of supply of demand. Who honestly believes there is a strict relation between the two? Who honestly believes that the prices of derivatives are the fair value of some lottery in the long run? Alternatively, who honestly believes they are the result of the subjective probability of some agent? Chances are that the best description of the market, when seen from above, is just chaos.

By contrast, I propose that the only way to look at the market, even to define it, is through the eyes of the market-maker of contingent claims, and through the material process he is engaged in – dynamic replication followed by recalibration.¹¹ A process, as I have said, that takes place

outside probability, yet brings order to the chaos. My criticism of probability applies both to objective probability (either in the frequentist or the propensity interpretation) and to subjective probability. What I mean by the ‘end of probability’ and its replacement by the price is just the end of the ‘state’. Even the single-case probability relies on the notion of possible state.

2.8 The market as the conversion of the image of thought

I should add that, in my metaphysical construction (or ‘system’, if I must speak like a speculative philosopher), the market is not even a picture; it is a logic. According to me, prices do not even occur inside a market or inside a picture, or a representation, or a theatre of the market. Rather, prices and the market occur ‘inside’ the contingent claim. They are embedded in it. They are an integral part of its *dynamics*, where the ‘dynamics of the contingent claim’ is not to be understood, of course, as an evolution of some kind within an external space – we are not talking, here, of the price dynamics, for that would be circular – but as the *dynamics of genesis*.

The metaphysical modality of possibility, usually associated with the fixed and identifiable states of the world, is perverse; it is not something lying ahead in the future, but in the past. It is a fabrication, a few items we fictitiously vary in the actual real and fictitiously project backwards (thus creating alternative possible worlds) in order to make it sound as if the real is the realization of a past possibility and *for this reason* is contingent. We then illegitimately project possibilities in the future. Possibility is as morbid as debt, because it is loan taken from the real.

I propose to transform, or convert, the whole picture. Instead of difference that is based on the recognition of identified states and the recognition of their difference, instead of difference that is derivative on identity, I propose a *difference without underlying states*, or a differential mark that has only itself to support itself – in other words, a sheet of paper on which the formula of difference is written: If A , Pay $x(A)$, Else if B , Pay $x(B)$, or again, the contingent claim. (You can take this phenomenon to be the definition of writing.)

Insomuch as possibility had only the possible state and its recognition to turn to, the contingent claim, this sheet of paper with the formula of *absolute difference* (that is to say, not derivative on underlying states) written on it, has the exchange to turn to. Although ‘ A ’ or ‘ B ’ sound like underlying states, I say the written formula is absolute and not relative to any states because I *literally* interpret ‘ A ’ or ‘ B ’ as written marks, not as states. They do not point to any existent metaphysical states. It is our classical image of thought that tends to represent them as states. States don’t mean anything unless involved in a picture where we recognize them, typically the picture of a tree where the *transition* between states is of essence. The sheet that supports the marks is materially one and not partitioned into states. As a matter of fact, it could burn (for instance) and disappear altogether.

What I am saying is that the classical image of thought needs the *transition* and invents the states for this reason. However, once writing is invented in replacement of representational thought, the transition is no longer required. We wake up the next day, *without a transition*, we read the formula that is written on the contingent claim, and we know what to do and what to pay or receive. All we know is that our world is contingent, that it could have been otherwise. We didn’t track the tree of

possibilities that has led us to this difference. The mere possibility that the sheet of paper on which the contingent claim is written might ultimately be destroyed is an a priori argument against the tree and the transition.

In the end, the exchange (or price) and the contingent claim sit at an opposite end relative to possibility and probability. I deduce the exchange, as a category of thought, in the same movement of conversion (or dynamics of genesis) as the contingent claim, without even talking of exchanging partners yet. From this, you can see how different from the standard conception my conception of price and exchange ultimately is. It is in a second instance that I remark that the exchange requires an exchange place where anybody can be.

Another consequence is that contingent claims admit of prices, not of expected values, and can never be redundant because in my converted picture there are no states of the world left anyway over which the notion of replication and redundancy can be defined.

The conversion takes place outside time and even 'before' time. It is the metaphysical operation which allows me to open the other door than the probabilistic one and to consider the *place process* instead of the temporal process. Conversion happens before time because it withholds time, and it is formal for this reason. At this level, it is purely a conversion of images of thought that we're talking about. It is only later, when we find that we have no possibility but to reintroduce time, that we will bring back the stochastic processes and their 'diagrams'. We have no choice but to project contingency over states all over again, no choice but to devise probability models. No choice but to replicate dynamically the contingent payoff. However, this brings in the market-maker who *can* subsequently invert the probabilistic model because he engages in dynamic replication. Indeed, my credo is that implied volatility makes sense only in the hands of the dynamic hedger of the option.¹² And this brings in recalibration, which is how probability is exceeded again and the original sense of the market (the one following from the conversion) is recovered.

2.9 Ascending to the metalogical level

Really, my purpose is to *deduce* the exchange. With the conversion, I am really taking my start from a primitive stage, where even the logic of exchanging is not yet operative. Reasoning metalogically, I identify this primitive stage as one of identity and convergence, or a stage of valuation, a ruinous picture indeed for the market and the exchange, and I propose that the conversion transforms this into the contingent claim, into difference and divergence, thereby creating the market in the process, as value is converted into price.

The strategic reason for my doing this is to distance the contingent claim and price from state, identity and probability as much as I can. I think my metalogical reasoning can even be pushed one level higher. If we view a mathematical function (say) as admitting of different values, that is, as differing, only on account of its underlying state variable; if, that is, we understand function value as following from the underlying state variable in that (causal) order of assignment, we speak of *valuation*; we value the function. If, by contrast, the logic of assignment is forgotten – imagine that the algorithm is lost; that the 'tables' are buried by some major earthquake or flood – and all that is

left is the function without the underlying state; if, in other words, only a fragment of the logic is left, the minimal part that says: ‘The value is Y , but it could have been different’, then we speak of *pricing*.

My real motivation is to release contingency from any idea of chronology, and correlatively, from the picture of the tree, the transition and the arborescence. The logic of assignment of value and of valuation (‘The value is Y because the state is X and $Y = f(X)$ ’) is chronological, whereas the logic of price (‘The value is Y , but it could have been different’) is independent of time. The contingency of an actual or even past thing and the contingency of a non-actual or future thing should be thought in the same way.

To price a contingent claim, we shouldn’t step backwards in time and reduce contingency to possible states in a tree. We should step backwards in another dimension than time. We step backwards in the market. The market is only ‘accidentally’ immersed in time. Likewise, Pierre Menard steps backwards in another dimension than chronology and when we observe that Cervantes’ *Quixote* is only chronologically, that is to say accidentally, anterior to Menard’s and that Menard is producing a serious task in his writing the *Quixote*, we realize that the accident of time is not serious. Chronological time is, indeed, an unavoidable accident. However, the genesis of the market is outside time. How we bring back the time-independence of the contingent claim and the time-independence of the market into the unavoidable temporal process is by dynamic replication, according to me.

Dynamic replication is what allows us to slide back in time the clause ‘The value is Y , but it could have been different’ without going through the tree. The reason why we invert the formulas of valuation (BSM or the like) against the market price, and the reason why the dynamic hedger alone is entitled to so inverting them, is precisely to be able to repeat, at every point of time, the final clause ‘The value is Y , but it could have been different’. Inverting the formulas of valuation is exactly repeating, before maturity, the inversion that we saw was operative when the valuation clause, ‘The value is Y , because $Y = f(X)$ ’, got transformed or inverted into the pricing clause: ‘The value is Y , but it could have been different’. To put it differently, recalibration is what brings back the market, what repeats it, and blows away the valuation tree at every point of time.