CHAPTER ELEVEN

Morphological Eidetics for a Phenomenology of Perception

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We tackle the problem of naturalizing phenomenology by proceeding from examples. These belong to the phenomenology of perception and bear on, first, the theory worked out by Husserl in the third logical investigation, which deals with the relation of dependence between spatial extension and sensible qualities; second, the links between geometry, vision, and kinesthetic control analyzed in *Ding und Raum*; and third, adumbrative perception as described in *Ideen* I. Geometrical models are worked out for each of these examples. They share rather sophisticated concepts of differential geometry, such as those of smooth manifold, fibration, calculus of variations, stratification, control, and singularity. Our main epistemological claim is that naturalizing phenomenology is an enterprise that has to be carried out in two steps, the first of which concerns the mathematical schematization of the eidetic descriptions, and the second, the implementation (e.g., neural) of the resulting algorithms.

1. INTRODUCTION

1.1. ORIENTATIONS FOR NATURALIZING PHENOMENOLOGY

I will try to tackle the difficult problem of naturalizing phenomenology by proceeding from technical examples.[1] To do so, I will adopt the following strategy.

1. The naturalization of phenomenology must be grounded in a *natural* science whose object is the phenomenal appearing itself, considered as a *natural* process.
2. The link between naturalist explanations, mathematical models, and computer simulations on the one hand, and phenomenological eidetic descriptions on the other, can be set up by viewing the latter as constraints on the former.
3. The key point is the mathematical schematization of phenomenological descriptive eidetics, that is, the elaboration of a *mathematical* descriptive eidetics (which Husserl thought impossible in principle).[2] For us, to naturalize an eidetics consists in implementing its mathematical schematization in natural substrates (e.g., neural nets).
4. This epistemological position can be summarized with two slogans: "Every description is the name of a problem." And, "Every concept is the name of an unknown algorithm."

1.2. EIDETICS OF SENSIBLE SCHEMATA

We chose our examples to illustrate different phenomenological problems and mathematical tools. They concern essentially the phenomenology of perception and the relationship between its morphological basis and the semantic levels of predication and judgment. It would be relevant here to give a detailed account of Husserl's concept of noema. But it would take too long. We refer only to the bibliographical items and to Rudolf Bernet's 1991 paper "Le Concept husserlien de noème." In Husserl's 1906–1908 lectures on the theory of knowledge and the theory of meaning and in *Ideen* I, Bernet distinguishes three dimensions of noematic ideality:

1. The noematic appearance:[3] the object as it is intuitively and immediately given (by direct acquaintance) in the constituting multiplicity of its adumbrations (*Abschattungen*). It is not itself an intentional meaning but a presentation, a display (*Darstellung*) of the object.[4] Its ideality is morphological.
2. The noematic meaning: a syntactically structured categorial content associated with judgment. Its ideality is logical.
3. The noema as object = *X*: of a constitutive rule, identity pole, or synthetic unity of appearances. Its ideality is transcendental.

It is, of course, essential not to confuse these three different dimensions. It would be an amphiboly in Kant's sense.

2. THE TRANSCENDENTAL TURN IN THE PHENOMENOLOGY OF PERCEPTION

Let us begin by reminding ourselves of certain aspects of the phenomenology of perception, e.g., in Husserl's 1907 lectures *Ding und Raum*.

2.1. PERCEPTION AND REDUCTION

In a transcendental perspective, the phenomenology of perception aims at clarifying the way the world of three-dimensional "things" constitutes itself as a transcendent world in the immanence of lived experiences and intentional acts.

In as much as the transcendental reduction brackets objective space and time, pure lived experiences can no longer be identified with psychological events (i.e., mental contents, acts and processes, or private feelings). Thus transcendental reduction reveals perceptual intentionality as a "constitution" of external objects.

The eidetic structures of perceptive experience are rooted in the rule-governed stream of consciousness. By its rules, the immanent temporal order of lived experiences constitutes objectivity. In that sense, the *passive synthesis* of the sensible *ante*-predicative manifestation is the ultimate ground of logical and predicative acts.

Such an analysis of perceptive evidence, and especially of the correlation between sensorial hyle and immanent lived experiences, on the one hand, and on the other, the objective properties of transcendent objects, raises a new de jure question: "How are evidential statements bearing on an objectivity which is not effectively given in the phenomena, possible?" (*Ding und Raum* [hereafter *DR*], §7, p. 19).

2.2. SELF-POSITING VERSUS EXPOSITING (DISPLAY), IMMANENCE VERSUS TRANSCENDENCE

The transcendental reduction reduces perception to immanent pure experience, which, contrary to things, states of affairs, and events of the external world, depends upon a consciousness of absolute givenness excluding any doubt and even any "doxa." This does not hold for the "exposition"—display, *Darstellung*—of a thing. The givenness in person (*Leibhaftigkeit*) of a thing does not possess any absolute character and is coupled with doxic modalities (*Glaubhaftigkeit* or propositional attitudes).

This opposition between "absolute data" (immanence) and "exposition" or "presentation" (transcendence) corresponds to that between *self-posited* immanent perceptions and presentational ones. Self-posited perceptions give their object in an *adequate* and *complete* way. This is not the case with presentational perceptions, which give only partial aspects (adumbrations) of the objects and need therefore an "*identification synthesis*" or a "*consciousness of identity*" unifying the different adumbrations as adumbrations of one and the same object (noema as object = X in the transcendental sense).

As the symbolic correlate of noetic syntheses, the object cannot therefore be a *real* component of the lived experiences adumbrating it. It can only be an *intentional* component. The givenness of an object is never absolute, never carried out "in the mode of a self-posited object" (*DR*, p. 30). Its essential inadequation and incompleteness constitute the *phenomenological origin of perceptive intentionality* as pointing toward an external world.

2.3. LIVED EXPERIENCES AND SPATIALITY

We will see that one of the main difficulties faced by Husserl was the impossibility of conceiving *geometrically* of a spatial extension different from objective physical space. Of course, every perception possesses an extensional moment, but "space is the necessary form of thingness, and not the form of 'sensible' experiences" (*DR*, §14, p. 43). For Husserl, space always belongs to transcendent objects. As the form of any external transcendence, it is adumbrated and constituted in specific experiences. But these are not themselves objectively spatial. How then to conceive of their "immanent" spatiality?

2.4. PRESENTATION (*DARSTELLUNG*) AND APPREHENSION (*AUFFASSUNG*)

The intentional relation of immanent perceptive contents to the transcendent objects they adumbrate depends upon their *apprehension. Auffassung* is a *process* operating on sense data: "It is by their apprehension that they [the sense data], which per se are dead stuff, acquire a sense which gives life to them, in such a way that an object can be displayed" (§15, p. 46).

We meet here the four main terms of the noetico-noematic correlation: hyletic data are animated by an "intentional morphe" (noetic syntheses = apprehension) that converts them into noematic appearances that adumbrate objects. Apprehension is always an *interpretation*. Husserl's concept of meaning is therefore an active one. It concerns effective cognitive algorithms processing hyletic data and converting them into presentational adumbrations.

Now, insofar as it is impossible to reduce the appearance of any object to one adumbration alone (it would have to be a complete givenness), there always exists a co-givenness of an infinite number of adumbrations of one and the same object. In that sense, the appearing always goes beyond what is intuitively given. It necessarily implies some reference to other, counterfactual possibilities of display.

2.5. EIDETIC CHARACTERS OF SENSIBLE SCHEMATA

Perception constitutes what Husserl calls a regional ontology. He considered it the most basic regional ontology, the one upon which the others are built up. In *Ideen* I and II, Husserl describes the sensible objects as "archi-objects" given "in person" pre-judicatively in an aesthesic noetic synthesis. This mode of appearing is not that of concrete material things, but that of sensible schemata constituted in the temporal flow of adumbrations.

There exist three main phenomenological characters of sensible schemata.

1. *The relation of foundation of sensible qualities in their spatio-temporal extension.* The spatiotemporal extension of a sensible schema—what Husserl calls its "spatial body"—constitutes its "characteristic eidetic attribute," "an eidetic form of all real properties." The spatial body is filled in by sensible qualities (colors, textures, roughness, etc.). It is an originary datum of any perceptive experience.
2. *The saliency of the form (the gestalt) so qualified.* It is necessary for catching the form and is realized through qualitative discontinuities.
3. *The adumbrative perception.* A sensible schema gives unity to different aspects. This adumbrative mode of manifestation is originally characteristic of perception and is "different in principle from the manifestation of real properties [material properties of things]" (*Ideen* II, §32, p. 131).

3. THE THIRD LOGICAL INVESTIGATION AND THE MORPHOLOGICAL ANALYSIS OF IMAGES

After these theoretical preliminaries we come to our first example. It concerns the naturalization of a very simple eidetic description, namely, the morphological description of a form, of a visual gestalt, given by Husserl in the first chapter of the third Logical Investigation. This text has been carefully analyzed by Kevin Mulligan, Barry Smith, and Peter Simons.[5] We also commented on it elsewhere.[6] It brings into action two fundamental gestaltist concepts, namely, that of merging (*Verschmelzung*) and that of segmentation (*Sonderung*), which are closely related to those of covering (*Überdeckung*) and fulfillment or filling-in (*Erfüllung*). In general, any mereological structuration of a whole in parts is the result of segmentation processes merging some moments and separating others.

3.1. THE HUSSERLIAN DESCRIPTION OF THE DEPENDENCE RELATION "QUALITY → EXTENSION"

Husserl begins to develop the key points in §4 of the third Logical Investigation, referring to Carl Stumpf's works.[7] As a "corporeal figure," the extension of an object is a primary quality filled in by sensible secondary qualities. As it is explained in *Ding und Raum*: "Any body, and more precisely any sensible schema of full corporeality, is a spatial corporeality (a spatial figure) 'over which' or 'in which' sensible qualities spread" (p. 297). Husserl distinguishes carefully a quality as *abstractum* and "the immediate moment which refers to it in intuition."[8] He calls the latter an "ultimate specific difference." The relation between a particular spatial extension and its immediate qualitative moments is a "functional dependency," whereas that between the kinds of extension and quality is an "eidetic law" that legalizes these functional dependencies.

Husserl then works out an *objective* and *a priori* conception of the dependence law: "Obviously, this is not a mere empirical fact, but an *a priori* necessity, which is grounded in pure essences [*in den reinen Wesen*] (*LU* III, §4, p. 237).

In §§5–7 this point is elaborated. The dependence/independence distinction is *objective*. It is given in an "apodictic evidence" and ruled by an "objective legality." It is free from any link to any effective psychological consciousness, to the "facticity of our subjective thought" (§6, p. 242). We meet here a typical example of a *synthetic a priori* principle.

3.2. ANALYSIS OF THE DESCRIPTION

Many other Husserlian texts, in particular *Ding und Raum* and *Ideen* I, deepen the analyses of the third Logical Investigation in what concerns the filling-in of spatial extension by sensible qualities.

3.2.1. Extension Is a Universal Topological Format for Qualities

Husserl emphasizes the fact that it is the cohesion of extension (its spatial "order," its topology) that confers unity upon qualities.

> Color data are not scattered and without connection. They share a fixed unity and a fixed form, the form of a pre-phenomenal spatiality. (*DR*, §21, p. 69)

> The privilege of the spatially distinctive mark is that its continuity corresponds to the continuity of qualities: different qualities (visual, tactile), lacking connections in themselves, receive from it their unity. (p. 346)

Spatiality is a universal format for sensible qualities.

3.2.2. The Primacy of Extension and the Dependency "Quality → Extension"

There exists, therefore, a primacy of extension: the dependence relation "quality → extension," even if bilateral (an extension can exist only if it is qualitatively fulfilled, and a quality can exist only if it fulfills some extension), is so fundamentally *asymmetric* that it can in fact be considered *unilateral*: "Places don't receive their order from colors, but, on the contrary, colors from places" (*DR*, p. 347). The spatial figure prescribes its rule for qualitative filling-in and for its variations according to "typical laws of filling-in." It is in this way that "pure spatiality" can be acquired "as a fundamental form of thingness" (§78, p. 264).

3.3. SEGMENTATION AND MERGING: CONTINUITY AND QUALITATIVE DISCONTINUITIES

The intuitive qualitative moments can be apprehended only if they form a global unity which stands out against a background (what Husserl calls a *phänomenale Abhebung*). In order to be grasped, a phenomenon must be "salient." How can such a saliency be obtained? To explain this key point, Husserl introduces, following Stumpf, "the difference between contents *intuitively 'separated'* from their neighbors, and contents *merged* with them" (*LU* III, §8, p. 247).

Merging (*Verschmelzung*) of neighboring contents yields an effect of wholeness,[9] a passage from local to global. On the contrary, segmentation (*Sonderung*) is an obstacle to merging and limits parts in wholes. Husserl emphasizes the fact that it is rooted in the concept of *discontinuity*: "Sonderung beruht . . . auf *Diskontinuität*." And he summarizes the process in the following way (his emphasis): "*Two simultaneous sensible concrete realities form necessarily an 'undifferentiated unity' when all the immediately constitutive moments of the first pass continuously over into the constitutive moments of the second*" (§8, p. 248).

Qualitative discontinuities are sharp transitions between ultimate specific differences. They are discontinuities of the functional dependency "quality → extension." They structure the covering (*Deckungszusammenhang*) of extension by quality. They can be grasped only if they are contiguously unfolded "against the background of a continuously varying moment, namely, the spatial and temporal one" (§9, p. 250). In short, spatio-temporal extension has to be the medium of a spreading (*Ausbreitung*) of qualities. It is from this spreading that Husserl derives the following definition of the gestaltist concept of qualitative discontinuity: "It is from a limit of space or time that one jumps from one visual quality to another. In this continuous transition from one part of space to another, one doesn't progress also continuously across the covering quality: at some point, neighboring qualities present a finite (and not too small) gap" (§9, p. 250).

And Husserl significantly adds that spatiality operates in this eidetic description not only as a "*sensible* moment" whose "objective apperception constitutes the phenomenal spatiality" but also as an "*intentional* moment" where the objective spatial figure is intuitively displayed. Space possesses, therefore, a noetic face (format of passive synthesis) and a noematic one (pure intuition in Kant's sense).

This analysis is made more precise in *Ding und Raum*. For objects to appear, some "lines of discontinuity" or some segmenting "lines of prephenomenal delimitation" must exist: "Without qualitative discontinuities separating it from its surroundings, no image can be salient, and focus attention" (*DR*, §53, p. 185).

Segmentatibility is a "characteristic" of the visual field closely related to its spatial "order" (its topology). It provides a pre-phenomenal mereology, that is, individuated two-dimensional constituents that can adumbrate three-dimensional objects.

3.4. THE CONFLICT BETWEEN THE SYNTHETIC-MATERIAL-MORPHOLOGICAL AND ANALYTIC-FORMAL-LOGICAL LEVELS

Morphological essences belong to the regional ontology of perception—which Husserl calls "material"—and not to formal ontology. The dependence law "quality → extension" is *synthetic a priori* and not analytic. One of the main problems is then to know what its *mathematical* status can be. Concerning this point, Husserl remains rather ambiguous. The reason is that, according to him, only analytic laws "can be completely 'formalized.'" And any analytic law, as he puts it in the third Logical Investigation, "is built up on formal logical categories and categorial forms" (§12, p. 260). Accordingly, in the second chapter of the same investigation, Husserl develops a general axiomatics of dependence relations.

To understand this crucial point, which engages the scientific destiny of phenomenology, we must take into account the Husserlian incompatibility between the "vague morphological essences" of sensible intuition and the mathematical idealities, especially the geometrical ones. For Husserl, the essences abstracted from intuitive data are not "exact and ideal concepts" and, therefore, cannot be mathematical:[10] "Essences apprehended by direct ideation [*Ideation*] in the intuitive data are 'inexact' and must not be confused with 'exact' essences, which come from a *sui generis* 'idealization' [*Idealisierung*]" (§9, p. 249).

According to Husserl, there cannot exist any morphological geometry and, therefore, any morphological schematism of the dependence relation "quality → extension" that could play the role of a transcendental schematism for the morphological synthetic a priori laws.

As we showed elsewhere,[11] this rejection of any morphological schematism is catastrophic for the scientific project of phenomenology because it leads to a drastic opposition between descriptive eidetics and science: "The descriptive concepts of any pure description, that is, of a description which immediately and faithfully conforms to intuition [*Anschauung*], and therefore of any phenomenological description, are for reasons of principle different from the determinant concepts of the objective sciences" (§9, p. 245).

3.5. THE MORPHOLOGICAL SCHEMATISM OF THE DEPENDENCY "QUALITY → EXTENSION"

This very elementary (but fundamental) example shows us what might be meant, from a cognitive point of view, by a naturalized phenomenology. The eidetic description we just sketched out is a noematic one. It concerns the simplest and most primitive component of sensible schemata. To "naturalize" it we apply the following strategy. First, we convert the phenomenological descriptive eidetics into a geometrical one. The geometrical schematization of synthetic a priori laws is the key to naturalization. It does indeed provide a non-naively formal version of noematics. Once we have worked this out, we have to implement it in natural processes, for example, in macrophysical theories of self-organized complex systems (the "external" side of naturalization) or neural nets (the "internal" side of naturalization).

The key Husserlian concepts that have to be geometrically schematized are essentially the following: (1) space-extension; (2) concrete quality / abstract quality, species, kind; (3) dependence/independence, inseparability/separability; (4) unilateral functional dependency "quality → extension," covering, filling-in; (5) merging / segmentation of neighboring qualities; (6) continuity/discontinuity; and (7) diffusion, spreading (*Ausbreitung*).

4. THE RELATION OF FOUNDATION "QUALITY → EXTENSION" AND THE CONCEPT OF FIBRATION

4.1. THE PRE-PHENOMENAL SPATIAL ORDER AND THE CONCEPT OF A SMOOTH MANIFOLD

To work out the geometric schematization, we must of course link the pre-phenomenal intuitive and continuous spatial order with a geometrically well-defined structural level. This cannot be the topological level, which is too soft: topologically continuous structures, such as fractals, can have an infinite internal complexity. Nor can it be the metrical level, which is too rigid and characteristic of objective physical space. We have therefore to assume that the relevant level is an intermediary one. We choose the *differentiable* one.

Such an hypothesis can be historically confirmed. Indeed, the Stumpfian concept of *Verschmelzung* used by Husserl comes from the German psychologist Johann Friedrich Herbart (1776–1841), who developed a *continuous* theory of mental representations. Essentially in the same vein as Charles Sanders Peirce after him, Herbart was convinced the mental contents are vague and can vary continuously. For him, a "serial form" (*Reihenform*) is a class of mental representations which undergo a graded fusion (*abgestufte Verschmelzung*), gluing them together through continuous transitions. He coined the neologism *synechology*[12] for his metaphysics (Peirce's neologism *synechism* is clearly similar). It is not sufficiently well known that Herbart's point of view greatly influenced Bernhard Riemann when he was elaborating his key concept of a Riemannian manifold (*Mannigfaltigkeit*).[13] Even if Riemann did not agree with Herbart's metaphysics, he stoutly proclaimed that he was "a Herbartian in psychology and epistemology." Erhard Scholtz has shown that in Riemann's celebrated *Über die Hypothesen, welche der Geometrie zu Grunde liegen* (1867) "the role of topological space [is] taken up in a vague sense by a Herbartian-type of 'serial form,' backed by mathematical intuition" (1992: 23). Now, the "intuitive" space underlying a Riemannian manifold is a smooth manifold. It must also be emphasized that the modern concept of a manifold was elaborated by Hermann Weyl, the great mathematical genius who was Husserl's philosophical disciple.[14]

4.2. THE CONCEPT OF FIBRATION

From our point of view, the schematization of the foundational law "quality → extension" has to correspond to a *category* (a type) of mathematical structure. This category must correspond to the "essences" governed by the law. And as for the specific mathematical structures belonging to it, they must correspond to models of phenomenal instantiations of the law (tokens).

We will show that the relevant category of structure is that of *fibration* or of *fibered space*.[15]

Intuitively, a fibration is a differentiable manifold E endowed with a *canonical projection* (a differentiable map) $\pi : E \to M$ over another manifold M. M is called the *base* of the fibration, and E its *total space*. The inverse images $E_x = \pi^{-1}(x)$ of the points $x \in W$ by π are called the *fibers* of the fibration. They are the subspaces of E that are projected to points in M.

In general, a fibration is required to satisfy the two following axioms:

(F_1) *All the fibers E_x are diffeomorphic with a typical fiber F.*

(F_2) *The projection π is locally trivial, that is, for every $x \in M$, there exists a neighborhood U of x such that the inverse image $E_U = \pi^{-1}(U)$ of U is diffeomorphic with the direct product $U \times F$ endowed with the canonical projection $U \times F \to U$, $(x, q) \to x$. (See Figures 11.1 and 11.2.)*

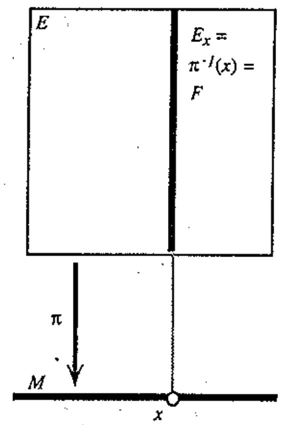

FIGURE 11.1. The structure of a fibration. M is the base space, E the total space, π the structural projection, $E_x = \pi^{-1}(x)$ the fiber over the point $x \in M$. All fibers are isomorphic to a typical fiber F.

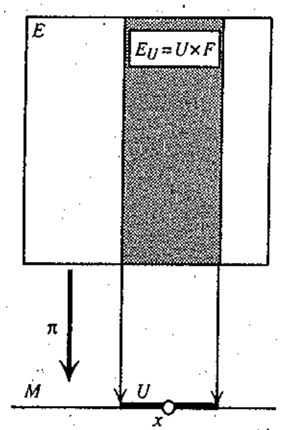

FIGURE 11.2. Local triviality of a fibration. Every point $x \in M$ of the base space possesses a neighborhood U such that $\pi^{-1}(U)$ is isomorphic to the trivial fibration $p: U \times F \to U$ (the projection of a Cartesian product onto its first factor).

In our case, the base manifold M is the ambient space of the substrate's extension W, and the fiber F the space G of the kind of sensible qualities under consideration (e.g., the color space). It is the canonical projection π that schematizes geometrically the law of foundation. It introduces a dissymmetry between the base M and the fiber F: the base is an "external" extensive

space, the fiber an "internal" intensive one. The very fact that π projects E on M expresses the unilateral dependency of the intensive magnitudes and secondary qualities relative to the extensive magnitude and the primary space quality: the external space *controls* the internal state.

Even if we cannot develop this point here, we must emphasize the fact that the concept of fibration is required by neurophysiological as well as by technological implementations. Indeed the orientation hypercolumns of the visual cortex (which associate each retinal position with the whole space of directions) implement a fibration.[16] It is the same for a computer screen: to each pixel ($w \in M$) are associated one to three bytes coding the gray levels or the RGB colors (fiber F).

4.3. THE CONCEPT OF A SECTION OF A FIBRATION AND HUSSERL'S FUNCTIONAL DEPENDENCIES

The general structure of a fibration allows for an easy mathematization of the Husserlian concept of functional dependency. Let $\pi : E \to M$ be a fibration and $U \subset M$ an open subset of M. A *section* s of π over U is a map $s : U \to E$ which *lifts* U to E. This means that s associates to every $x \in U$ an element $s(x)$ of the fiber E_x over x. We get therefore for the composed map $\pi \circ s : U \to E$ the identity of M. If there exists on U a local trivialization $\pi_U : E_U \cong F \times U \to U$, it transforms s in a classical map $x \to [f(x), x]$ of U into the direct product $F \times U$, where $f: U \to F$ is a map (a "function") of U into the fiber F. Therefore, the concept of section generalizes the classical concept of map, that is, of functional dependence. In general s is supposed to be continuous, differentiable, or analytic. It can present discontinuities along a singular locus. (See Figures 11.3 and 11.4.) It is conventional to write $\Gamma(U)$ for the set of sections of π over U.

A section of a fibration therefore exactly expresses a specific functional dependency of the qualitative moments (the fiber) relative to the extension (the base). We therefore retrieve exactly the "pure" Husserlian description and thereby "prove" that it is indeed mathematizable.

4.4. MORPHOLOGIES AND QUALITATIVE DISCONTINUITIES: THE WAY FROM HUSSERL TO THOM

Qualitative discontinuities that make a phenomenon salient are discontinuities of sections. Let $q_1, \ldots, q_n$ be the sensible qualities that can fill in the external space W. They belong to space kinds $G_1, \ldots, G_n$ (colors, textures, etc.). Let $s_1(w), \ldots, s_n(w)$ be the sections expressing the filling-in of W by the q_i (that is, $s_i : W \to E_i$ is a section of the fibration $\pi_i : E_i \to W$ with fiber G_i). This allows us to schematize the concept of phenomenological

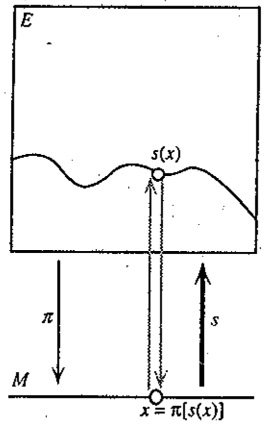

FIGURE 11.3. The concept of a section s of a fibration π. The section s associates to every point x of $U \neq M$ a point of E lying over x that is a point of the fiber E_x. Therefore, $\pi[s(x)] = x$.

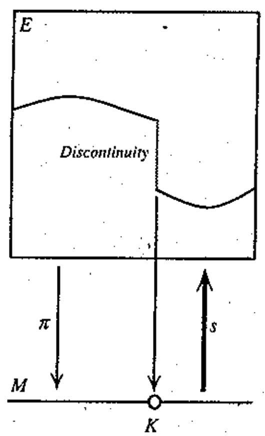

FIGURE 11.4. A section of a fibration can be discontinuous along a singular locus K.

saliency constitutive of the concept of morphology. One of René Thom's major merits was his understanding of this point.[17] Thom calls a point $w \in W$ where all the sections $s_i(w)$ are locally continuous a *regular* point. Regular points form an open subset R of W.[18] Let K be the complementary closed set $K = W - R$. K is constituted by special points—called *singular*—where at least one of the sections $q_i(w)$ is discontinuous. It realizes a morphology whose phenomenological saliency corresponds exactly to Husserl's pure description.

4.5. THE CATEGORIZATION OF THE QUANTITATIVE SPACES

It must be added that qualitative kinds G are generally categorized in different species ("essences"). This means that (1) the space G is decomposed in domains (categories) $p_1, \ldots, p_k$ by a system of boundaries Σ (this is called a *stratification*), and (2) there exist in these domains *prototypes* corresponding to "central values."[19]

The existence of such categorizations entails extra qualitative discontinuities. In addition to the discontinuities proceeding from those of the sections $s_i(w)$, there exist those due to the fact that the values $s_i(w)$ change category. They can exist even if the sections s_i are continuous. (See Figure 11.5.)

4.6. MORPHOLOGICAL SCHEMATISM AND DESCRIPTIVE EIDETICS

Morphological schematism allows us to mathematize all eidetic components of the phenomenological description.

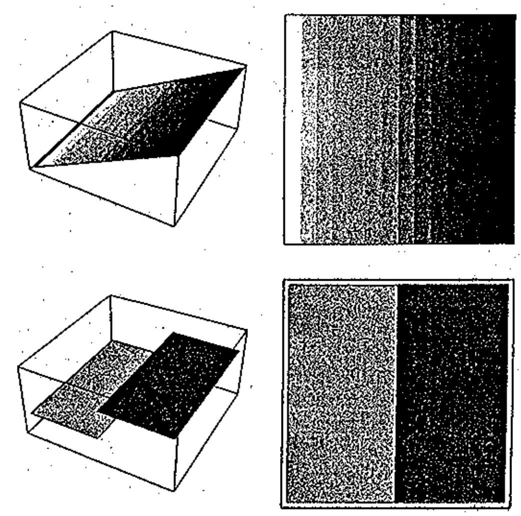

FIGURE 11.5. When the fiber of a fibration is categorized (here a "continuum" of gray levels is categorized into two discrete values) a continuous section becomes a locally constant one presenting qualitative discontinuities.

1. The functional dependencies determined at the level of minimal specific differences correspond exactly to *particular* sections $s : W \rightarrow E$ of *particular* fibrations $\pi : E \rightarrow W$.
2. The qualitative salient discontinuities are modeled by discontinuities of sections $s \in \Gamma(U)$.
3. The eidetic law "concretely determined by its material contents" corresponds to a particular fibration $\pi : E \rightarrow W$ of fiber G but without any particular given section. π implicitly contains an infinite universe of potential functional dependencies, namely, all the sets of sections $\Gamma(U)$ for $U \subset W \subset M$. Such a fibration models a first level of abstraction: the global extension M and the qualitative kind G are determined, but not the particular domain W or the section s.
4. The synthetic a priori law of dependence "quality $\rightarrow$ extension" corresponds to the general mathematical structure of a fibration. It concerns the most abstract kinds—the essences—of space and quality (this is a second level of abstraction).
5. Last but not least, the "analytic axiomatization" of this synthetic dependence law in the framework of formal ontology corresponds to the *axiomatics* of fibrations, that is, to the *category* (in the mathematical sense) of fibrations.

4.7. THE PROBLEM OF ABSTRACTING QUALITIES

It is possible to confirm the phenomenological relevance of such a "fibrational" schematization by looking at the way it solves the difficult Husserlian problem of abstracting qualities from pre-empirical coverings.

At a (naive) level, one could consider that a "simple" (i.e., constant, uniform) color such as "red" is a common quality shared by all red things. But even if traditional, this naively extensional point of view is not convincing. First, it takes for granted the "atomist" nominalist axiom of the primacy of individuated things. Now, things result from highly complex noetico-noematic processes of constitution and are by no means primitive data. Second, it does not take into account the fact that, in a covering, quality can vary continuously. Hence the question: "Isn't the general use of the concept of quality made possible only by an idealizing concept formation?" (*DR*, p. 363).

Husserl's answer is quite subtle. It is based on the concept of *localization*, that is, of restricting coverings to "small pieces of the surface": one divides the extension into domains where the covering quality can be considered "simple": "Every coloration which is not the extension of a simple quality is decomposed by a space partition into partial colorations of different simple qualities" (p. 363). If the coloration varies a lot, then one must take the *limit* of a decomposition into small domains, and "the simple quality becomes the filling-in of a spatial point" (p. 363).

4.8. SIMPLE QUALITIES AS GERMS OF SECTIONS

We will see later (in section 6.2) that for Husserl the points of the prephenomenal spatial order are not ideal mathematical ones, and are only defined at a certain scale or resolution. But let us suppose for the moment that they are limits of smaller and smaller domains. The problem is the following.

If we admit that the fiber G of the fibration $\pi : E \rightarrow M$ *is already known*, and if we consider a section $s \in \Gamma(U)$ over U, then we can of course consider its value $s(w)$ at every point w of U. In particular, for every element p of G—namely, for every "simple" quality—we can consider the *constant* section s_p defined by $s_p(w) = p$ for every $w \in U$. The problem is that knowledge of G *presupposes* the idealizing abstraction of qualities. This confusion between what is constituting and what is constituted is one of the worst phenomenological mistakes.

We must therefore suppose that *we don't already know* G and dispose only of coverings, that is, sections $s \in \Gamma(U)$. How it is then possible to reconstruct G from these data alone?

The first idea is precisely that of localization, that is, of the *restriction* of a section defined over an (open) set U to an (open) subset V of U. Restrictions are transitive. Husserl was aware of this fact. After having explained how spatial continuity confers its unity upon qualitative determinations (see section 4.3), he claims, "But all this is still not enough, and hides something" (*DR*, p. 346), and refines his description: "Color covers extension and forms or orders itself in it. To every fragment of extension there corresponds a fragment of coloration, and also again to each fragment of fragment. And all the partial colorations are ordered in the unity of the global coloration, whose form of order is precisely the global extension" (p. 346).

Through localizations and restrictions, it is possible to shift from the global level to the local one. Reciprocally, it is possible to shift from the local level to the global one by *gluing* local restrictions.

The second main idea for reconstructing the fiber G only from sections is to take the limit of localizations whose domain becomes "point-like." It is Husserl's idea to define a "simple quality" as "a filling-in of a spatial point."

Technically, let $s \in \Gamma(U)$ be a section of π over U. To define its value at $w \in U$ *purely in terms of sections*, we look at the (open) subsets V of U containing w. They form a structure known as a *filter*, and it is possible to take the *limit* of the restrictions of s relative to this filter. In this way we get what is called the *germ* of the section s at w. It localizes s to an "infinitesimal" neighborhood of w. And Husserl's eidetic description has been retrieved.

5. PHYSICO-MATHEMATICAL MODELS OF THE THIRD LOGICAL INVESTIGATION

Let us now go on to the possibility of naturalizing this geometric schematization of Husserl's eidetic description by using physico-mathematical models. We will sketch out two examples. They identify the *Verschmelzung* and *Sonderung* noetic syntheses with effective algorithms processing signals (hyletic data).

5.1. THE MUMFORD-SHAH MODEL

One of the main problems of natural and computational vision is to understand how signals can be transformed into geometrically well-behaved observables. The key point is the segmentation process. Let therefore $I(x, y)$ be a rough image. It is an unstructured hyletic datum without any geometrical structure. The question is: how can we go from it to a morphologically organized perceptual image? What "geometry machine" provides its morphological formatting?

More than a thousand segmentation algorithms have been recently worked out which merge local data in homogeneous regular regions and separate the regions by regular crisp edges. The main problem is that two-dimensional regions and one-dimensional edges are *in competition* and, moreover, are geometrical entities of *different dimensions* which interact very subtly. In fact, underlying this proliferation of models there exists a deep unity. As was emphasized by Jean-Michel Morel, "Most segmentation algorithms try to minimize . . . one and the same *segmentation energy*" (Morel and Solimini 1995). Such segmentation energy allows us to compare one segmentation with another and to measure how well it approximates the rough signal. The most popular model for doing so is the Mumford-Shah (David Mumford won a Fields Medal in algebraic geometry and became a specialist in the study of vision).

In a recent paper, "Bayesian Rationale for the Variational Formulation" (Romeny 1994), David Mumford emphasizes that "one of the primary goals of low-level vision is to segment the domain W of an image I into the parts W_i on which distinct surface patches, belonging to distinct objects in the

but what is w?

scene, are visible." The mathematical problem is to use low-level cues "for *splitting* and *merging* different parts of the domain W."

In these models, there exist *two parts*: a *prior* model and a *data* model. Actually, the prior model takes as an a priori the phenomenological *Verschmelzung / Sonderung* complementarity (even if there exists of course no reference to phenomenology in computational vision). It imposes the constraint of approximating the signal I by piecewise smooth functions u on W–K, which are discontinuous along the regular edges K.

Now, we pass from the eidetic description to a true mathematical modeling by introducing a way of *selecting*, from among all the allowed approximations (u, K) of I, the best possible one. For this, Mumford used a *functional* $E(u, K)$, which allows the comparison between two segmentations $E(u_1, K_1)$ and $E(u_2, K_2)$. E must contain at least three terms: (1) a term that measures the variation and controls the smoothness of u on the connected components of W–K; (2) one that controls the quality of the approximation of u by I; and (3) one that controls the length, the smoothness, the parsimony, and the location of the boundaries, and inhibits the spurious phenomenon of oversegmentation.

The "energy" E proposed by Mumford and Shah (1989) is:

$$E(u, K) = \int_{W-K} |\nabla u|^2 dx + \int_W (u - I)^2 dx + \int_K d\sigma$$

Minimizing E is a compromise among the following:

1. The homogeneity of u on the connected components of W–K: if $u = cst$ then $\nabla u = 0$ and $\int |\nabla u|^2 dx = 0$, and therefore minimizing this term forces u to be as constant as possible.
2. The approximation of I by u: if $u = I$ then $\int (u - I)^2 dx = 0$, and therefore minimizing this term forces u to be as close as possible to I.
3. The parsimony and the regularity of the boundaries: they are measured by the global length L of K, $L = \int_K d\sigma$, and therefore minimizing this term avoids oversegmentation.

The model can be interpreted probabilistically, using the equivalence $E(u, K) = -\log[p(u, K)]$, p being a probability defined on the space of possible segmentations (u, K).

Such a variational algorithm optimizes the way in which neighboring pixels can be merged in homogeneous regions separated by qualitative discontinuities. It provides, therefore, a *variational* approach to Husserl's *Verschmelzung / Sonderung* duality. It transforms the segmentation problem into a particular case of what is called in physics a "free boundary problem." It is an extremely difficult problem; in fact, it is not yet completely solved.

5.2. MULTISCALE ANALYSIS AND ANISOTROPIC DIFFUSION EQUATIONS

The Mumford-Shah model is variational and global. It is possible to construct other models of the *Verschmelzung / Sonderung* complementarity which are more local and based on anisotropic nonlinear partial differential equations.

The first move consists in interpreting *Verschmelzung* as a *regularization* or smoothing process, which filters the signal by convoluting it with Gaussian profiles. The different widths of the Gaussian profiles correspond to different scales. Such a process is more plausible neurophysiologically, for it is well known that there exist, in the visual system, fields of neurons whose receptive fields are Gaussians. But Gaussians being the kernel of the heat equation, it is possible to interpret the *Verschmelzung* process as the application of a *diffusion equation* to hyletic data in a scale space.[20] The equation is

$$\partial_s I_s = \Delta I_s = \mathrm{div}(\nabla I_s)$$

where I_s is the image blurred at scale s, ∂_s the partial derivative of I relative to s, Δ the Laplace operator, div the divergence, and ∇ the gradient operator.

The problem is that the heat equation is isotropic and homogeneous and cannot, therefore, be *morphological*. It is adapted to the regularization of the image (*Verschmelzung*) but not to its segmentation (*Sonderung*). It also blurs the boundaries. To be morphological, a diffusion equation must preserve boundaries from blurring. Indeed, it should even *enhance* them. This may seem quite impossible because merging and splitting are opposite processes. But it is nevertheless possible if the diffusion equation is *anisotropic*.

Several solutions to this problem have been worked out. The initial key idea was contributed by Jitendra Malik and Pietro Perona and consists in inhibiting the diffusion near the boundaries, that is, in zones of large gradients. It leads to equations of the type

$$\partial_s I_s = \mathrm{div}[g(\nabla I_s) \cdot \nabla I_s]$$

where g is a decreasing positive function such that $g(x) \underset{x\to\infty}{\longrightarrow} 0$.

But the most radical solution is to use a diffusion equation that inhibits the diffusion transversely to the level lines of the image. We get (P.-L. Lions, J.-M. Morel, L. Alvarez) the equation

$$\partial_s I_s = \Delta I_s - \frac{H(\nabla I_s, \nabla I_s)}{|\nabla I_s|^2}$$

where H is the Hessian of I_s (the quadratic form given by its second partial derivatives).[21] It is uniformly parabolic along the level curves of I but totally degenerate in the gradient direction. It makes the level curves evolve as fronts with a normal velocity equal to their curvature (it is the problem known as "curve shortening," "flow by curvature," and "heat flow on isometric immersions").

This example makes still clearer what might be meant by the naturalization of phenomenology (see section 3.5). The eidetic description provides synthetic a priori laws of perception as the foundation law "quality → extension," the complementarity *Verschmelzung / Sonderung*, and so on. These laws shift from a descriptive status to an explicative one by being translated as structural design constraints for effectively implementable algorithms. In this way, we get physico-mathematical models that are naturalist models of noetic synthesis operating on hyletic data and generating a noematic structuring of the phenomena (in this case morphological segmentations).

6. GEOMETRY AND VISION IN *DING UND RAUM*

6.1. THE TWO-DIMENSIONAL SPATIAL ORDER OF THE VISUAL FIELD

Let us come now to our second example. It concerns Husserl's description of the two-dimensional spatial order of the oculomotor field and of its kinesthetic control.

6.1.1. The Basic Character of the Visual Field

Husserl devotes extremely precise analyses to the visual field M considered as a pre-empirical extension endowed with a spatial order. This field is basic because all the adumbrative contents of things are fragments of it. It raises (at least) four questions concerning (1) its structure; (2) its kinesthetic control; (3) the phenomenological constitution of the global space "co-perceived" in every perception (it is the phenomenological origin of transcendental aesthetics); and (4) the constitution of "the astonishingly separated position of the ego" as a correlate of the surrounding world. This ego and its lived body allow us to convert pure lived experiences into "internal," psychological, and neurally implemented ones.

6.1.2. The Finiteness of the Field and Space as Manifold (*Mannigfaltigkeit*)

The field's extension is *finite*. This has two main consequences: first, the impossibility of a complete and adequate presentation of a thing (we have already seen that this lies at the root of perceptual intentionality); and second, the synthetic a priori necessity of gluing different fillings-in of the field in order to constitute the temporal flow of perceptive adumbrations in a global space.

The field is structured by the *fixed order* of its positions: "Visual space is the idea of a two-dimensional manifold consisting, to a certain extent, in pure fixed points" (*DR*, p. 313). Its position in the field is therefore a fundamental determination of any element of sensation. It provides its "qualitative particularity" and converts it into what Hermann Lotze called a "local sign" (p. 364).

But the different positions are all equivalent and can be commuted: "The visual field is a two-dimensional manifold which is congruent with itself, continuous, simply homogeneous, finite and, of course, delimited" (§48, p. 165). This is an excellent picture of the characteristic properties of the visual field as a two-dimensional manifold endowed with its group of automorphisms. To summarize:

1. It is a metrically bounded Riemannian manifold (its finiteness is what is called a compacity property).
2. It is a multiscale two-dimensional manifold with varying resolution, fine-grained at its center and coarse-grained at its periphery.[22]
3. It is a homogeneous space whose group of automorphisms acts transitively (it can exchange any two points).

6.1.3. Dimensionality and Stratification

It is well known that it is extremely difficult to define correctly the concept of dimensionality. The standard mathematical definitions are not phenomenologically relevant, for they are abstract and ideal, not intuitive. Phenomenologically, the only intuitive data are fragments of coverings of extension by qualities. Just as Husserl resorted to the iteration of fragmentation to define what can be a "simple" quality (see section 4.5), he resorts to a new remarkable geometrical intuition to define dimension. "Bi-dimensionality means that every fragment of the field is delimited by dependent limits, which themselves are in turn fragmentable continuous manifolds. . . . But now the limits are no longer fragmentable, they are simple elements of extension, namely, 'points'" (*DR*, §48, p. 166). Since then, this intuition has been formalized, essentially by Hassler Whitney and René Thom, under the name of *stratification*.

Let us consider a domain D of M (an open, connected, and simply connected subset of M equal to the interior of its closure). D has the same dimensionality as M. It has a *boundary* $B = \partial D$. Husserl implicitly assumes that the boundaries are regular (piecewise smooth submanifolds). His first fundamental intuition is that the boundary of a manifold of dimension n is of dimension $n - 1$, that is, that the boundary operation ∂ drops the dimension by 1. His second intuition is that it is possible to iterate the operation ∂ and to consider $\partial^k D$ for $k = 1, 2, \ldots$. His third intuition is that points are nonfragmentable, without boundary, and therefore of dimension 0.

These three key intuitions allow us to define without difficulty the dimension of M. One considers all the possible domains $D \subset M$ and computes the minimal number k of ∂-operations necessary to reach points; k is the "phenomenological" dimension of M.

6.2. The Concept of Multiscale Field

Another fundamental Husserlian intuition concerns the multiscale character of the visual field.

6.2.1. Visual Atoms and Scale Invariance

With regard to the concept of fragmentation, Husserl analyzes the phenomenological relevance of the concept of "point." The question is to know what "*visual atoms*" are and, more precisely, "whether fragmentation, which leads to *minima visibilia*, provides essentially ultimate elements in this way, and if points are therefore the same thing as visual atoms" (*DR*, §48, p. 166). Husserl goes thoroughly into this intuition by emphasizing even the self-similarity and the scale invariance of the pre-phenomenal continuum. He notes "the essential similarity to itself of the visual field, on a large and small scale" and explains that "it is obviously this immanent similarity which, as evident generic similarity, justifies the transposition of the eidetic relationships discovered, so to speak, in the macroscopic universe, to the microscopic 'atoms' situated beyond divisibility" (§48, p. 166).

Husserl therefore introduces the idea that for "visual atoms" a base scale can be established. We find the very same idea in recent theories of vision with the implementation of resolution in the receptive fields of retinal ganglionary cells. Husserl also introduces the idea (largely confirmed by the work on multiscale differential geometry) that classical geometry is an *ideal-*

ization transposing to an infinitesimal scale the eidetic structures drawn from the perceptive scales.

6.2.2. CHANGES OF SCALE AND INTENTIONALITY

Husserl also uses the concept of scale when he analyzes the fact that the field is not really translation-invariant, the images located at its periphery being "less differentiated" than those located at its center (*DR*, §55, p. 193). Resolution is weak (coarse-grained) at the periphery and strong (fine-grained) in the foveal zone. Peripheral images share a "poorer internal differentiation" and "less and less separated parts will be extracted" (ibid.). On the contrary, central images share a "richer" internal differentiation.

In image processing and the neurophysiology of vision, this corresponds to filtering techniques by "pyramidal" (or multiscale) algorithms. One smoothes the optical signal by convoluting it with (for example, Gaussian) filters (see section 5.2). If I_0 is the initial signal (the brute image, the "hyle"), and if one smoothes it at scale s, one gets an image I_s. If $s' > s$, $I_{s'}$ is "poorer" and I_s "richer." One shifts from I_s to $I_{s'}$ by convolution (this is very easy) and, reciprocally, from $I_{s'}$ to I_s by deconvolution (this is very difficult).

This idea is explicit in Husserl. Focalization—a cognitive process linked with attention and interest—allows us to shift continuously from the "poorest" images to the "richest" ones. We will see later that Husserl conceives of the *identifications* between the different parts of the field (when its contents change according to its kinesthetic controls) in terms of "intentional rays" "going through" the images and their presentational contents. He very accurately describes the processing of scale change. Passing to a finer scale enriches the presentational content. This is an "ex-plication" of the image, an increase of differentiation. When a "poor" image differentiates itself into a "richer" one, "every intentional ray divides . . . itself according to these internal differences, and develops itself [*explizierend*]" (§55, p. 193). Such an "ex-plication" reaches a *maximum* in the central region of finest scale. Maximally "ex-plicated" intentions "don't admit any supplementary ex-plication" (p. 195).

Reciprocally, when a "rich" image becomes "poorer," it "de-differentiates" itself. Its presentational capacity becomes weaker. Intentionality no longer "ex-plicates" but "im-plicates" itself.

Husserl introduces here a new key idea, that of a *retention* (a sort of short-term memory) of the "rich" image in the "poor" one. What is distinctly apprehended through attentional focusing "is intentionaly maintained and serves to enrich implication" (*DR*, p. 340). For Husserl, it is because apprehension is susceptible to such "enrichments" and "ex-

plications" that it is necessarily intentional. The "rich" retained intention "permeates" "the series of poorer images from which the movement of images flows" (*DR*, §55, p. 194). Accordingly, "the differentiated detail means . . . more than what it displays" (§55, p. 195). Exposition is no longer fully explicit, but in part implicit. We meet here a new aspect of this characteristic of intentionality that a perception never reduces to a mere presentational display and is always co-given with an infinite nexus of systematically connected presentations. Not only does there exist for every transcendent object a continuous infinity of co-given adumbrations, but for even a single adumbration there exists a continuous scale of ex-plication. Every object is noematically the unity of an infinity of multiscale aspects, and the fact that infinitely many different images are in fact co-given in any single image implies an intensional (in the pragmatic sense), symbolic, and indexical structure of any perceptual display. The latter is richer than the strict display. It contains potential dimensions to which it refers in a no-longer immediately intuitive but mediately *symbolic* way.

In that sense, there always exists a "semiotic" dimension in perception, each actual perceptual display pointing as a sign—as a Peircean index—toward a potential infinity of other actualizable ones.

6.3. THE CONSTITUTIVE ROLE OF KINESTHESES

6.3.1. The Primacy of the Continuum and the Kinetic Synthesis

For Husserl, the problem of the kinesthetic control of perception belongs to "the great task . . . of penetrating as deeply as possible into three-dimensional phenomenological 'creation,' or, in other words, into the phenomenological constitution of the identity of the body of a thing through the multiplicity of its appearances" (*DR*, §44, p. 154). According to him, it would be a "monstrous presumption" to believe there exist simple answers to these problems, for "[they] count among the most difficult questions bearing upon human knowledge" (§44, p. 156).

"The effective identity of the appearing object" cannot result from isolated presentations. As a "meaning identity" linking the different perceptions adumbrating a single thing, it is of a logical nature. But it presupposes the possibility of flowing *continuously* from one adumbration to another: "It is only when, in the unity of experience, the continuous flow from one perception to another is warranted, that we can speak of the evidence according to which an identity is given" (§44, p. 155). In other words, *logical identity depends upon continuous variability*, and according to Husserl, this is the very origin of synthetic a priori laws. The logical synthesis of identity presupposes continuous synthesis, which is the only kind that corresponds to originary intuition. But in so far as continuous synthesis is always temporal, it is in fact a *kinetic* synthesis. It rules phenomenologically temporal series corresponding to three classes of movements, namely, those of the eyes, the body, and objects.

6.3.2. The Kinesthetic Sensations

But here again, it is essential not to confuse the constituting level with the constituted one, and, adopting the natural attitude, consider that the movement of external objects is the most primitive one. For it too results from a constitutional process. In a correct eidetic description, the phenomenological source of movement has to be found not outside but *within* the internal *kinesthetic sensations*.

Husserl starts from the evidence that "the extensional moment of visual sensation . . . is not sufficient for making possible the constitution of spatiality" (*DR*, §46, p. 160). Kinesthetic sensations are absolutely required. They share a very peculiar status. Indeed, they belong to the "animating" apprehension (the intentional morphe) resulting from noetic syntheses, but without possessing a proper presentational function: they make possible the presentation of external objects without being themselves presentational. We will see that they act as *controls* on the visual field.

Besides their "objectivizing" function, kinesthetic sensations share a "subjectivizing" function that lets the lived body appear as a proprioceptive embodiment of pure experiences, and the adumbrations as subjective events.[23]

6.3.3. The Control Space of Kinestheses and Its Paths

The space $\mathcal{K}$ of kinesthetic controls is multidimensional and hierarchized. It includes varying degrees of freedom for movements of eyes, head, and body. These controls exert their "objectivizing" function through *temporal paths* k_t in $\mathcal{K}$. They "build continuous unities only in the form of paths, where a linear [i.e., one-dimensional] submanifold of the global manifold of kinesthetic sensations [$\mathcal{K}$], superimposes itself, as a filling-in continuum, on the continuous unity of the pre-empirical temporal path" (*DR*, §49, p. 170).

6.3.4. The Kinesthetic Control and the Relativity of Movement

There exists an obvious equivalence between a situation where the eyes move and the objects in the visual field remain at rest, and the reciprocal situation where the eyes remain at rest and the objects move. But this trivial aspect of the relativity principle is by no means phenomenologically trivial, at least if one does not confuse what is constituting and what is constituted. Relativity presupposes an *already* constituted space. At the pre-empirical constituting level, one must be able to discriminate the two equivalent situations. The kinesthetic control paths are essential for achieving such a task.

6.3.5. The Kinesthetic Control Is Not Directly an Associative Link

Husserl starts with the simplest situation where, the body being fixed and the objects remaining at rest, the control reduces to the correspondance $k \leftrightarrow i$ between the purely ocular kinesthetic sensation and the visual image i. A temporal series $t_1, \ldots, t_n$ entails a series of correspondances $k_1 \leftrightarrow i_1, \ldots, k_n \leftrightarrow i_n$, and a temporal flow, a continuous correspondance $k_t \leftrightarrow i_t$.

Husserl then analyzes the nature of the dependence relation $k \leftrightarrow i$ and shows that it cannot be an associative one (in the sense of reinforcement learning). Obviously, a control k cannot determine any particular image i. Any control k can be linked with any image i. There exists therefore no association, no "empirical motivation," between k and i. What can be the nature of the link?

6.3.6. Kinesthetic Controls Act as Operators on Paths

Husserl's idea is the following. Let us consider an initial link $k' \leftrightarrow i'$ and another image i'' (compatible with i'). To each visual path i_t going from i' to i'' corresponds a kinesthetic path k_t going from k' to another control k'', which will settle a new (final) link $k'' \leftrightarrow i''$. In other words, "associated with the representation of an image movement $i' \leftrightarrow i''$, I immediately get the representation of a kinesthetic flow $k' \leftrightarrow k''$, as a flow belonging to it" (§51, p. 179). This eidetic description can be modeled in the following way. Let $\mathcal{F}$ be the "space" of visual images. Ideally (that is, if one doesn't take into account the pixelization of images), it is a functional space of infinite dimension. $\mathcal{K}$ *controls* $\mathcal{F}$ in the sense that the *paths* in $\mathcal{K}$ operate on the *paths* in $\mathcal{F}$. More precisely, let $k \leftrightarrow i$ be an initial link and k_t a path with origin k in $\mathcal{K}$ (an ocular movement). The k_t determines a movement in $\mathcal{F}$ (an image movement) i_t and, moreover, with the same temporal parametrization. This correlation is sufficiently strong to allow the visual system to solve the *inverse problem*, namely, to retrieve the path k_t from the given path i_t.

But this clarification is not sufficient. The movements i_t are variations of the contingent filling-in of the visual field M. And the problem is to understand the nature of the link between $\mathcal{K}$ and M themselves. There exists a "fixed association," not between any $k \in \mathcal{K}$ and any $i \in \mathcal{T}$ but between M and $\mathcal{K}$ *as such*, that is, between "the entire extension" and "k in general." And Husserl asks "how to understand and describe more exactly the phenomenological situation" (§51, p. 180) constituted on the one hand by the fixed association $M \leftrightarrow \mathcal{K}$ and, on the other hand, by the nonassociative correlations $k_t \leftrightarrow i_t$?

6.3.7. Kinesthetic Controls Are Gluing Operators

Husserl discusses the elementary example of a square S with corners a, b, c, d. His description can be formalized in the following way. Let us suppose for simplicity that the field M is a simple domain D (a disk). To focus on the point a means that D is centered on a. (We also assume that the square is sufficiently small relative to D; if this is not the case we have to consider sufficiently many intermediate points). (See Figure 11.6.)

To each position $p = a, b, c, d$ corresponds a token D_p of the field D. And if the figure i_a filling in D_a can "refer" to the figure i_b filling in D_b, it is because D_a and D_b *overlap*, and are *glued* together through their intersection $U_{ab} = D_a \cap D_b$. This means that there exists a local gluing isomorphism ϕ_{ab} : $U_{ab} \subset D_a \rightarrow U_{ab} \subset D_b$ identifying the intersection U_{ab} viewed as a subdomain of D_a with the same U_{ab} viewed as a subdomain of D_b. In the continuous limit, there exists a temporal series D_t with gluing operators $\phi_{tt'}$ for t and t' sufficiently near. This spatiotemporal series is filled in by the image series i_t.

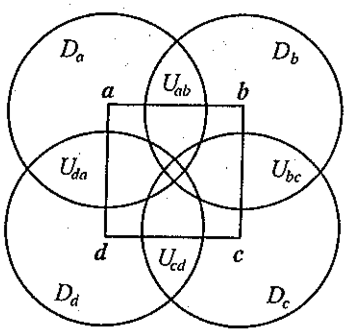

FIGURE 11.6. Scanning square S with corners a, b, c, d. To each position p corresponds a token D_p of the visual field D centered on p (focalization on p). The neighboring D_p overlap.

To say that the "pointing" of each i_t to other $i_{t'}$ is *intentional*, or that intentions "go through" the series i_t, is to say that intentionality corresponds to gluing operators identifying different points of the visual flow as the same. This interpretation will be confirmed later (in section 6.3.9) when we will see Husserl introducing "intentional rays going through the images" to identify the corresponding points of different images (point tracking).

More precisely, intentionality corresponds to the realization in consciousness of the gluing operators.[24] Once again, it is essential here not to confuse, as the natural attitude does, the constituting level and the constituted one. For the latter there exists a priori an ambient global space where the continuous series of the D_t and of the fillings-in i_t are embedded. *But in absence of such a space, only the D_t and the gluing operators $\phi_{tt'}$ remain*, and one shifts from the local level to the global one by gluing local charts all identical to D. The gluing operators must therefore be realized in consciousness. This is the main role of kinesthetic controls: *the k_t are gluing protocols.*

We have therefore to distinguish three levels: (1) kinesthetic controls k_t, which are paths in $\mathcal{K}$; (2) manifolds resulting from gluing identical D_t according to the k_t (namely, by identifying corresponding points through "intentional rays going through the images"); and (3) qualitative fillings-in i_t of the D_t.

It is through the D_t that the k_t control the i_t and $\mathcal{K}$ controls $\mathcal{T}$. The temporal paths in $\mathcal{K}$ act on $\mathcal{T}$. Let $k_0 \leftrightarrow i_0$ be an initial link and let us consider a path k_t in $\mathcal{K}$ with origin k_0. k_t acts as a gluing protocol for the flow D_t of the visual field. D_0 is filled in by i_0, D_t by i_t, and the gluing of the D_t induces the gluing of the i_t in a "glanced over" global image. The spatial gluing of the i_t, and their intentional referring presupposes a temporal gluing, an immediate memory, a "retention" of i_t in $i_{t'}$, for t' sufficiently close to t.

6.3.8. The Categorization of $\mathcal{T}$-Paths by $\mathcal{K}$-Paths

Such a schematization (which Husserl could have gotten from Hermann Weyl if he had been less dogmatic) allows us to clarify one of the most difficult paragraphs of *Ding und Raum*, namely §52, where Husserl tries to make more explicit the control $\mathcal{K} \leftrightarrow M$ (M is the true visual field, and D is a simplified version as a two-dimensional disk). First, as we already seen, no association $k \rightarrow i$ can exist between a particular kinesthetic sensation and a particular image. On the other hand, insofar as the pre-empirical field M is a system of fixed places, it cannot admit any association $k \leftrightarrow M$ either. The manifold M "doesn't provide any base for a link [$k \leftrightarrow i$]. For it is precisely always there, always given with any k and therefore with all possible k and

k-paths" (§52, p. 183). The solution to this difficulty proposed by Husserl is quite subtle: "The link can consist only in the *formal unity* of paths" (our emphasis). This means that the paths i_t of $\mathcal{T}$ are grouped in *equivalence classes* $[i_t]$ (that is, categories), which Husserl calls *path types*. It is only with an image path *type* that a determined kinesthetic path can be associated. Between a path type $[i_t]$ and a kinesthetic path k_t an associative link can exist—and does exist effectively—that is, a true control (a motivation). It is the "form"—the type—of paths in $\mathcal{T}$ that is associated with paths in $\mathcal{K}$.

Husserl therefore introduces the profound and beautiful idea that the paths k_t in $\mathcal{K}$ *categorize* the space of paths i_t in $\mathcal{T}$. But this means exactly that the paths k_t act on the paths i_t. Let i_0 be an initial image: k_t defines—through the gluing protocol for the D_t—a path type $[i_t]$, and the image path i_t is the token with origin i_0.

Moreover, we even find in *Ding und Raum* the idea that the image flows i_t are in fact *trajectories of vector fields* in the functional space of images $\mathcal{T}$. Besides the retention in every i_t of the $i_{t'}$ preceding it (for $t' < t$ sufficiently close to t), there exists also a *protention* of i_t in the successive $i_{t'}$ (for $t' > t$ again sufficiently close to t), what Husserl called an "infinitesimal deformation" of i_t. Husserl had a clear idea of what is called a tangent vector to a manifold, since he spoke of "linear direction of change" and of "differential."

6.3.9. Transversal Intentionality and Intentional Rays: From Identification to Identity

We have seen that a thing (at rest) is constituted through "an ideal system of possible continuous series of appearances temporally coinciding with possible, continuous, and motivating kinesthetic series" (*DR*, §55, p. 190). These series are the trajectories $k_t \leftrightarrow i_t$, and the correlated thing is provided by the *unity* of their flow. Now, unity is based on an "intention-toward" (the protention along the temporal trajectory), which is not "delimited": "By virtue of its essence, it doesn't terminate in the determined image. . . . It goes through it and keeps this character of a transversal element, independently of the way the continuity of appearances can extend" (§55, p. 190).

But what can the nature of this "transversal" intentionality be? It is here that Husserl introduces the concept of *intentional ray*. As he puts it: "Intentional rays which go through the actually given images . . . link, in a consciousness of unity, corresponding points of continuously transformed images" (§55, p. 191). Intentional rays process what can be called a "point tracking." They are gluing isomorphisms embedding a part of i_t in $i_{t'}$, Husserl speaks even of a "'mono-univocal' correspondance" (p. 198), what is now called an injection (or a monomorphism). Identity consciousness is based on such operations of identification: "Points located on the same intentional ray display . . . one and the same objective point" (§56, p. 198). Husserl stresses this fundamental fact: "A consciousness setting unit here goes through the pre-empirical temporal continuity, [and] a flow of contents, tracked by the intentional ray, displays, step by step, the same thinglike point" (§56, p. 198).

We show in Figure 11.7 how a global image I with global extension W can be constituted via a window D (a square for simplicity) moving diago-

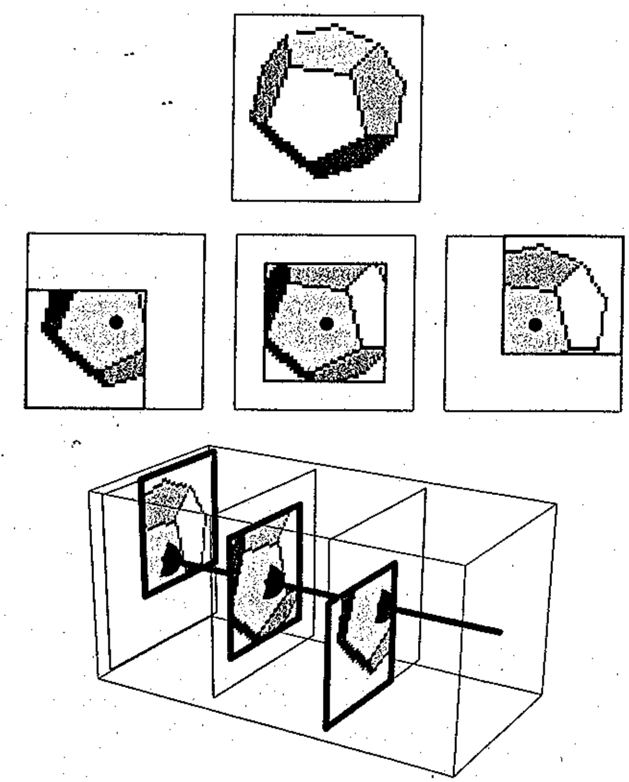

FIGURE 11.7. Constitution of a global image I (a dodecahedron) in a global window W by moving a sub-window D (a square) in W. *Top*, I in W. *Center*, evolution of D. A point p occupies different locations in the different tokens of D. *Bottom*, an intentional ray tracks p and transforms it into an individuated objective "thing-like" point.

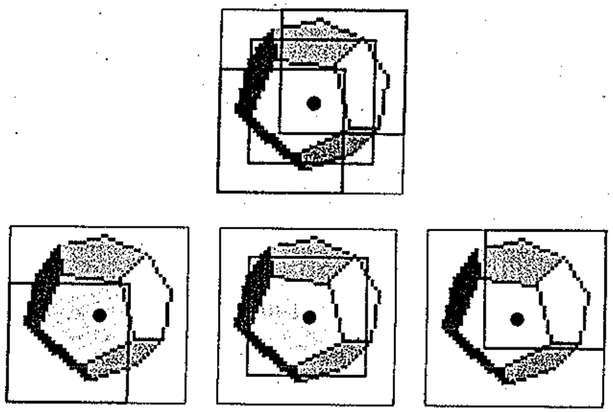

FIGURE 11.8. Once the global image *I* is constituted, we can think of the visual field *D* as a window moving in an objective external ambient space *W*.

nally in *W*.[25] In the first image we see *I* in *W*. In the second image we see how *D* evolves. We show in fact the filling-in of three exemplars of *D*. We show also a distinct point *p* occupying three different locations in the three exemplars of *D*. They represent the "immanent" local data whose gluing produces the "objective" extension *W*. In the third image we show the intentional ray tracking *p* and transforming it into an individuated objective "thing-like" point. In Figure 11.8 we show how, once the objective image has been constituted, it becomes possible to think of the visual field as a window moving in *W*.

We grasp very well here the necessarily *noematic* essence of objects as identity poles: a trajectory of identified points tracked from image to image corresponds to one and the same objective point. The condition of the possibility of any transcendent object is that "every point, in its relative place, is set up as an identical [unit] and intentionally maintained as an identical [unit]" (§61, p. 218). We witness here the genesis and the emergence of transcendence out of immanent acts and contents.

6.4. ELEMENTS FOR THE CONSTITUTION OF OBJECTIVE REALITY

Supplied with these basic eidetic descriptions, Husserl tackles the different components of objective reality. We have no room here to discuss this phase of his work. We will simply note that he begins with the constitution of objective *temporality*. That each point possesses its "pre-empirical temporal place" in an image path i_t implies that the identical objective (thing-like) point associated with a trajectory (a "transversal intentional ray") becomes an *individuated* point *temporally parametrized*. In other words, *once constituted, the noematic identity of the objects*, the trajectories of identification yielded by noetic syntheses, can be interpreted, according to the natural attitude, *as objectively temporal trajectories of individuated objective points*. So, temporality radically changes its status. It shifts from the lived pre-empirical immanence to the objective transcendence it displays.

Husserl also discusses the constitutive role of the different levels of kinesthetic control. Though already rather rich, the previous description concerned only the most elementary case of an object at rest and a kinesthetic control restricted to the oculomotor level. But there exist several coupled kinesthetic systems: eyes, head, body. Moreover, external objects can present different forms of movement: they can move across the visual field, they can approach or recede, they can rotate, and so on. This fact notably complexifies the relations between *k*-controls and *i*-displays but doesn't fundamentally change the principles of analysis.

Husserl also investigates binocular vision and stereopsis, with regard particularly to the constitution of objective three-dimensional space and the three-dimensional objects embedded in it. He emphasizes that stereopsis results from the fact that the two binocular images are not perfectly identical. Their slight discrepancies are *interpreted* by the visual system as relief cues and "depth values" (p. 173) superposed upon qualitative discontinuities.

Here again, Husserl's description is astonishingly modern. Pre-empirical (immanent, noetic) depth has nothing to do with a supplementary third dimension. But it *displays* a noematic properties of objective spatiality, namely the third dimension: "These differences of relief are not at first differences of place, but *they acquire the meaning* of differences of place" (§49, p. 174, our emphasis). It is apprehension (as interpretation and intentional morphe) that after having computed the tiny gaps between the two binocular images, constitutes, constructs cognitively, infers, the third objective dimension.

Studies on stereopsis, from Bela Julesz's pioneering works to Ninio's self-stereograms[26] show that this pure eidetic description is in perfect agreement with the present results of visual cognition.

To bring these few remarks on *Ding und Raum* to a close, we will underline the manner in which Husserl connects the typification of displaying paths (noesis) with objective ontology (noema). The main idea is that *ontological* characters of external objects are correlated to kinesthetically motivated *types* of possible chains of adumbrations. This point is crucial for the naturalization of the phenomenology of perception.

On the one hand, Husserl stresses the fact that the geometrical concepts of point, line, surface, place, figure, size, and so on, used in eidetic descriptions *are not spatial* "in the thing-like sense," since what is at stake is precisely the phenomenological genesis of the objective space. In particular, it is "senseless" to believe that "the visual field is . . . in any way a surface in objective space" (§48, p. 166), that is, to act "as if the oculomotor field were located, as a surface, in the space of things" (§67, p. 236).

But on the other hand, Husserl also explains that the kinesthetic complex, which with its coupled levels, allows us to access the infinite space and an "integral" (but always inadequate) display of the objects, "makes the oculomotor field (eventually enlarged to infinity) the mere projection of a three-dimensional spatial thingness" (§63, p. 227). In other words, once the three-dimensional global space R^3 has been constituted (through two-dimensional gluings and stereopsis), *one can reverse the dependence order* between what is constituting and what is constituted, and act as if (1) the visual field M were embedded as a fragment of surface in R^3; and (2) the filling-in adumbrations were projections on M of objects in R^3, and were therefore *causally* generated by them. As a result, what comes first in the order of phenomenological constitution becomes second in the order of objective causality, and vice versa. We meet here the main example of the inversion of priorities between the phenomenological approach and the objectivist one in the natural attitude. We witness the genesis of objective ontology.

Among all possible paths i_t, only those which are kinesthetically motivated are really possible (Husserl calls them "real possibles"). But this extremely strong constraint imposed by the *kinesthetic control* can also be expressed by *global geometric constraints* on images. The "computation" processed on the i_t through the "circumscribed lawfulness" constraining them, can also be executed from geometrical *transcendent* hypotheses setting a three-dimensional space, external objects, planes and directions of projection, and projections. That is why one can posit spatial three-dimensional objectivity and express the lawfulness of adumbrative flows *as if* it were a consequence. Frontal translations, closeness/remoteness, rotations, changes of orientation, occlusions, and so on, are object transformations that are *encoded* in typical transformations of images. But they are exactly the same as if they had proceeded from projections of Euclidean automorphisms of R^3.

In short, the typification of the temporal flow of paths i_t is actually imposed by the kinesthetic control (the motivation). But it is *also* describable in terms of objective transformations (rotations, etc.) of the noematic invariants extracted from the flow. In that sense, it determines a "thing-like" ontology of external objects.

We see up to what point objectivity is here *transcendental* and not naively ontological. It is the intentional (noematic) *correlate* of eidetico-constitutive rules operating noetically on hyletic data formatted by a pre-empirical transcendental aesthetics.

Let us stress again that an inversion of perspective becomes possible once three-dimensional space and three-dimensional objects become available as constituted entities. One can interpret in an objectivist way the continuous transformation i_t of two-dimensional images as if the body were embedded in objective space R^3, as if the oculomotor field were a piece of surface defined by the position of the body, and as if three-dimensional objects were projected onto it. Husserl claims that "from a formal point of view," these transformations are "exactly" the same as for "projections of a geometrical body on a plane" (§69, p. 241). This claim is essential, for it shows that "from a formal point of view"—that is, from the point of view of *an appropriate geometrical eidetics*—there exists an *equivalence* between the phenomenological-constitutive and the causalist-objectivist approaches. Sharing a *common* formal level, these two apparently opposed perspectives are in fact deeply *complementary*.

We therefore measure the crucial importance of a pure *geometrical* descriptive eidetics. Neutral as to the phenomenology of appearances and the ontology of things, it can be realized in *two* complementary ways: (1) cognitively, and in that case the problems of implementing noetic synthesis will have priority; and (2) objectively, and in that case the problems of physical causality will have priority. In this sense, it is the key to the naturalization of the phenomenology of perception.

7. ADUMBRATIVE PERCEPTION

7.1. IMPORTANCE OF ADUMBRATIVE PERCEPTION

A third remarkable example where Husserl's eidetic description is akin to contemporary cognitive research is that of adumbrative perception. We have already evoked it several times.

In the different elements of *Abschattungslehre* scattered throughout his works, Husserl investigates several problems whose contemporary relevance is striking.

1. The enigmatic *equivalence* between the temporal flow of adumbrations and the object = X as an intentional unity and noematic pole. We have seen that there exist eidetic "chaining rules," that is, a "descriptive composition" and an "internal organization" *prescriptive* for experiences. Such rules are correlative of the noematic unity of the object. They rule its appearing, its mode of display.

2. The incompleteness of adumbrations implies that the complete determination of object givenness can be only *temporal*. Incompleteness and temporality are essentially linked and manifest our cognitive finitude. Constituted in the immanent temporality of the pure Ego, the flow of adumbrations unfolds a dynamic order. Whence the key problematic of *anticipation*. The possibility of ruled coherent anticipations is one of the main characteristics of objective transcendence.
3. The incompleteness of adumbrations also generates a *horizon of co-givenness*: an actual adumbration is inseparable from an infinity of other potential and virtual ones.
4. Incompleteness generates finally the gap between intuition and intention. It produces the intentional meaning of the noematic object. As was stressed by Ronald McIntyre, it is essentially because the complete determination of givenness can only be temporal and grounded in anticipations that noematic rules are *semantic*. This attests to the phenomenological genesis of sense and denotation.

7.2. FLOW OF ADUMBRATIONS AND TEMPORAL SYNTHESIS

7.2.1. Adumbrations and Inadequate Givenness

Each perceptual display giving the object "properly" and "in person" is related to a horizon of other possible "improper" (unfit to match the hyletic data but counterfactually actualizable) displays lacking any presentational content. Each perceptual display is therefore necessarily embedded in a temporal flow of adumbrations evolving continuously "in a merged [*verschmolzen*] unity" out of which noeses extract intentional noematic identities. In every adumbration there is more than the adumbration itself. There is also the referring to (pointing toward) other co-given potential adumbrations.

Improper adumbrations are not intuitively presented but "symbolically" represented. This certainly does not mean that they are imaginary. Imagination is also an adumbrative faculty. The problem is that symbolic pointing is a constitutive part of the immediacy of intuition. The givenness of an object "in person" always acts as a *sign* as well. The direct acquaintance in perceptual evidence is in fact essentially indirect: "It refers, *thanks to its sense*, to possibilities of filling-in, to a continuous-unitary chaining of appearances" (*DR*, §35, p. 124, our emphasis).

7.2.2. Anticipations and Misapprehensions

The essential incompleteness and inadequacy of adumbrative perception implies the a priori synthetic necessity of ruled anticipations. But in-

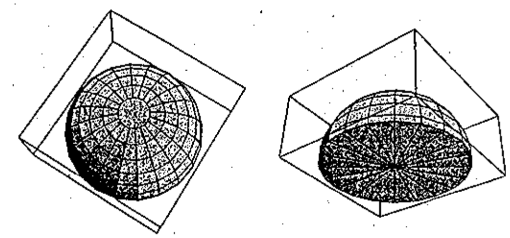

FIGURE 11.9. The problem of generic anticipations. They are nonmonotonic. When we see a spherical form from one vantage point, we anticipate seeing a spherical form from all vantage points. But of course that is not necessarily the case.

sofar as the other co-given adumbrations of the object correspond to non-intuitively filled-in intentions, anticipation can only be an expectation (and not an inference). This means two things.

First, anticipation will be only "*generic*." It will concern only the most probable and typical expectations. If we look from one viewpoint at what appears to be a sphere (Locke's example used by Husserl), we will assume that it is effectively a uniform sphere. But of course this is not necessarily the case. It could for instance only be a hemisphere. (See Figure 11.9.)

Second, mismatch is therefore possible and anticipations can fail. Generic anticipations are nonmonotonic (in the sense of default logics).

However, even if a fully determined anticipation is impossible, every anticipation remains submitted to the "circumscription," legalizing a priori the concept of an object. A mismatched anticipated adumbration can only be replaced by another *compatible* one.

7.2.3. Generic Anticipations and Typical Adumbrations

In *Ding und Raum*, Husserl analyzes in great detail *generic* anticipations. We have seen that in a flow of adumbrations, there exists a "protentional" "differential of movement" (that is, tangent vectors) defining anticipations and expectations. But among all the possible adumbrations of an object, certain are privileged and display the object in an optimal way.

Husserl introduces the profound idea that the "fullness" of presentation has a "degree." Not all adumbrations are informationally equivalent. The optimal ones are those of maximal degree: "The increase of filling-in or completeness of givenness comes to an end at limit points . . . where the increase becomes a decrease" (§32, p. 107). These "turning points" are "maximal points" of the "fullness of data." Protention (pointing toward) is not only a tangent vector (a "differential of movement") along an adumbrative path A_t. It is also a tendency toward the maximal informational completion.

Everything happens as if one could define, on the functional space $\mathcal{A}$ of adumbrations, a sort of *potential function* F (as in the classical calculus of variations)[27] measuring their informational content (their "degree"), a function whose maxima would select the optimal adumbrations. The idea is that there exists a *variational principle* governing the *anticipated* adumbrative flows (and not of course the effective flow, which is determined by the kinesthetic controls). The best adumbrations then act as *prototypes*, as "*attractors*" attracting the paths A_t.

Husserl is perfectly aware of the mathematical fact that the maxima of F must be *critical points*. We have seen that he speaks of "turning points where the increase becomes a decrease." They produce "preferential" prototypical (but still inadequate) displays. And Husserl concludes that adumbrations "*are themselves adumbrated* in this new sense" (§35, p. 125; our emphasis). The idea is astonishing. These *second-order* adumbrations correspond, in the realm of immanent experiences, to "the consciousness of givenness in its own and par excellence." It is toward them "that tend, in a way, every other display, every other consciousness of givenness, . . . the *intention* in the flow of perceptions" (§36, pp. 125–26).

According to Husserl, there exists therefore a *double* dynamic in the space $\mathcal{A}$. In so far as adumbrations are images ($\mathcal{A} \subset \mathcal{F}$), there exists the kinesthetic control $\mathcal{K}$ whose paths act on paths in $\mathcal{A}$ (see section 6.3). But insofar as each adumbration points spontaneously (without any k-control) toward an optimal filling-in, there exists also an optimization dynamic X on $\mathcal{A}$. X is the gradient dynamic associated with the potential F measuring the informational content (the "degree") of the elements $A \in \mathcal{A}$. The trajectories of X "climb up" F to its maxima (prototypical adumbrations), and $\mathcal{A}$ is so categorized in attractor basins. According to such a variational principle, perceptual intentionality is also always intentional in the sense of *finality*. As is stressed by Husserl, "this consciousness of [maximal] givenness . . . is the end of perceptive movement" (§36, p. 226).

Once again, we verify the extent to which Husserlian intentionality depends upon a continuous synthetic synthesis and a functional geometrodynamical schematism that has absolutely nothing to do with a logical theory of denotation.

7.2.4. Structural Stability and Relevance

Husserl introduces two other important ideas concerning prototypical adumbrations.

The first one is very akin to that of *structural stability*, pervasive in contemporary differential geometry thanks to great mathematicians such as Whitney, Thom, Smale, and Arnold. The prototypical adumbrations (maximal points) are structurally stable—or *generic*—in that they can vary "inside some limits" without altering the apprehension of the object: "noticeable" differences in the appearances are nevertheless informationally "insignificant": "The visual intention enters the privileged zone, but for the filling-in of this intention, every appearance that does not exceed some limit of variation is suitable" (§36, p. 128). In other words, there exists around each maximal point a central domain of strong stability where the quantitative discrepancies are qualitatively (morphologically) negligible.

7.2.5. Typicality, Relevance, Interest, and Conceptualization

Husserl specifies also his idea of "fullness of givenness." The potential F measuring the "fullness of data" (the informational content) depends upon the interest, the relevance, and the attention at stake in apprehension: "The spheres of completeness, the maximal points and zones, do not belong to the essence of appearance as such, but to the interest grounded in it and to the afferent attention" (§37, p. 130). For instance, in the perception of an object there will be an optimal face, an optimal distance, and so on. These optima are preferential in the sense of the modern theory of *prototypes* and *categories*.

Husserl then stresses the fact that this theory of optimal adumbrations provides the ground for "the empirical forming of concepts." *Conceptual generalization is a complex cognitive process rooted in the synthetic a priori laws of perceptual adumbrations.* It is a generalization through typicality. An empirical concept is no longer logically defined by its extension, but functionally as a *type*, as a *schema* in Kant's sense: "The way the concept is oriented is determined by the interest which rules the process of concept formation, the constitution of the consciousness of generality" (§37, p. 130). This very modern approach to concept formation is all the more remarkable, since it was elaborated in a period when the logicist's extensional conception prevailed.

7.3. GEOMETRICAL EIDETICS OF APPARENT CONTOURS

For a long time, I have stressed the link between Husserl's *Abschattungslehre* and many works in differential geometry and computational vision. The case is particularly clear in what concerns the key problem of outlines or *apparent contours* (ACs).

We must first use multiscale segmentation techniques (see section 5.2) for extracting from the signal geometrically well-behaved boundaries. Then we must use stereopsis and other cues (e.g., shading) to interpret these boundaries as ACs of three-dimensional, external objects. Let us suppose that these (difficult) tasks are already achieved. The remaining problem is then one of an essentially geometrical nature.

Geometrically, the Husserlian problem of the equivalence between a transcendent object T embedded in three-dimensional ambient space R^3 and the family (the functional space) of its adumbrating ACs is an absolutely nontrivial *inverse problem*. The direct problem is: given $T \subset R^3$ and a projection π (that is, a plane of projection P and a direction of projection δ), build the AC $AC_T(\pi)$ of T to π. It is already nontrivial and requires sophisticated mathematical concepts. Indeed an AC is the projection on P of the *singular locus* S of the restriction of π to T. S is the locus of points x of T where the direction δ is tangent to T and $AC_T(\pi)$ is the projection $\pi(S)$ of S.

In Figure 11.10, we represent in perspective the singular locus S of a horizontal torus T for a direction making a $\pi/12$ angle with the horizontal plane. We represent S alone, and S on the torus. In Figure 11.11 we also represent four adumbrations of T. Finally, in Figure 11.12, we represent eight ACs of T along a rotation path. The key feature is that between the second and the third steps, the AC presents two symmetrical exemplars of a singularity known as a swallowtail, whose unfolding creates two cusp points and a crossing point. We see perfectly well on this example what the optimal AC type is. It is the most complex one in the sense of singularity theory (Figures 11.11 and 11.12.)

Thanks to singularity theory, it is possible to classify the singularities appearing *generically* in ACs. They can visually be detected through fields of neural cells sharing appropriate receptive profiles.

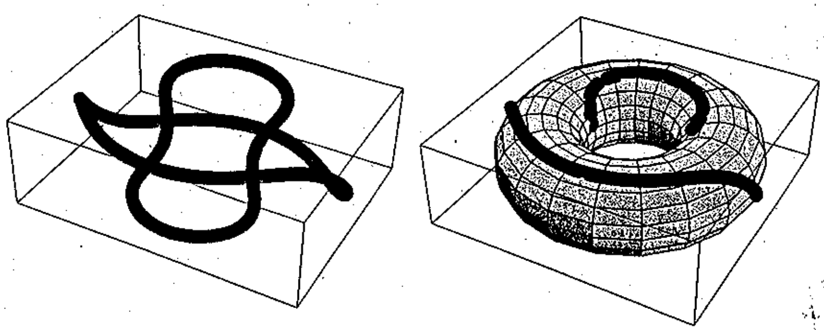

FIGURE 11.10. The singular locus S of an horizontal torus T for a direction making a $\pi/12$ angle with the horizontal plane. *Left*, S alone. *Right*, S on the torus.

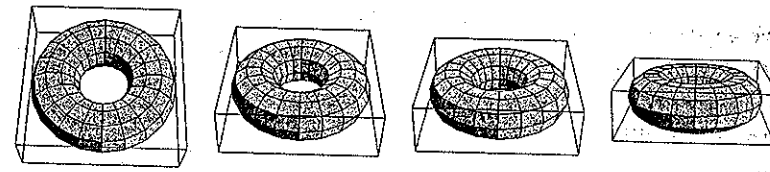

FIGURE 11.11. Four adumbrations of a torus T.

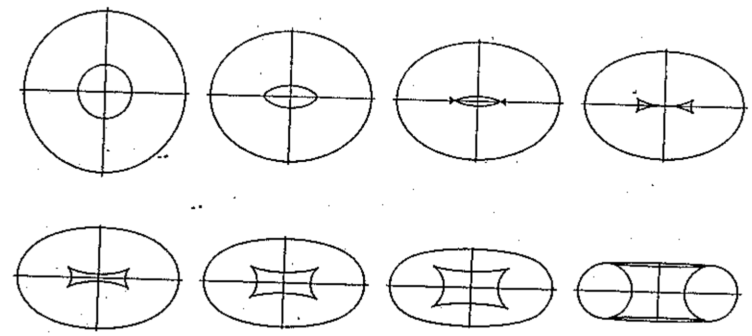

FIGURE 11.12. Eight apparent contours (ACs) of a torus T along a rotation path. Between the second and third steps, the AC presents two symmetrical swallowtail singularities whose unfolding creates four cusp points and two crossing points.

The situation becomes particularly interesting when singularity theory is coupled with the Riemannian geometry of the surface T: elliptic zones (where the points are like those of an ellipsoid), hyperbolic ones (whose points are saddle-like) with their two fields of asymptotic directions, and the parabolic curves separating these two kinds of zones. In accordance with the position of the direction of projection δ relative to these elements, one gets different singularities. For instance, the swallowtails of a torus correspond to the points where δ is tangent to an asymptotic curve at an inflection point (what is called a flecnodal point). The most complex case is that of "Godron points" where δ is a double asymptotic direction tangent to a parabolic curve.

If we consider now the space V of projections π, we can analyze the temporal evolution of the ACs of T along a path in V. This evolution depends on how V is decomposed into domains, each of which corresponds to ACs of the same qualitative type. What is called an *aspect* is a generic central value of a qualitative type. It is exactly what Husserl called a typical adumbration.

We see that the shape of T *categorizes* the space V of viewpoints in categories, each grouping the tokens of the same type of ACs. This categorization of adumbrations in aspects is realized by a system of interfaces K_T whose combinatorial version is called the *aspect graph* of T: G_T. There exists an *equivalence* between T and K_T. T allows us to determine K_T: it is the (easy) direct problem. Reciprocally, K_T allows us to reconstruct T: it is the (difficult) inverse problem.

The geometric equivalence $T \Leftrightarrow G_T$ is the mathematical version of the Husserlian eidetico-constitutive law. It can be formulated in terms of temporal flows by exploring the categorization K_T through *temporal paths* in V. This conversion of a "synchronic" structure into a "diachronic" one is the root of perceptive intentionality. It generates the belief in external reality.

We see the duality correlating the object = X as a noematic pole of identity and the adumbrations displaying the different aspects of the object. Every AC has a noetic face (the associated information processing) and a noematic face (its geometry). It is a noematic *appearance* and not a noematic *meaning*, a presentational content and not a propositional one. It can be conceived of in two complementary ways. If one considers it as a projection of an already constituted three-dimensional object embedded in objective space (see section 6.4), then it is an objective datum informing the cognitive system about the external world. In that case it depends upon a causal theory of reference and is a *large* content. On the other hand, if one considers it as an element of a system of other co-given ACs, it is a *narrow* content, functionally determined by its relations with them.

The link between the two perspectives is given by the object = X acting as a coherence and identity principle for the considered ACs, warranting a solution to the inverse problem because the aspect graph can be generated by a possible object. The object = X can never be displayed in perception. It acts semantically as a denoting symbol, but in a nontrivial way. In fact, it acts *in an intensional, indexical, and pragmatic way*, as a *choice operator* selecting counterfactually an aspect and actualizing it among others, which remain virtual (horizon of co-givenness).

We arrive therefore at the conclusion that there exists what Husserl calls a perceptive "*genealogy*" of denotation. It is only at higher cognitive levels, when objects have been converted into symbols that denotation in its classical logical sense recovers its rights. In the simplest act of denotation, information is processed in a twofold manner: morphologically (as adumbrations, incompleteness, anticipations, and so on) and symbolically (as denotation).

CONCLUSION

It is impossible to tackle here the many other correlated problems. We take the liberty of referring the reader to our other works for the following problems:

1. That of the natural *mereology* of spatial domains filled in by sensible qualities (namely, of sections of fibrations).[28]
2. That of the links between the descriptive eidetics of perceptive morphologies and the syntactico-semantic theory of perceptive *judgments*. Using deep links between formal logic and fibrations and more generally *sheaves* (what is called the *theory of topoi*), it is possible to formalize the *perceptual genealogy of predication* brilliantly worked out by Husserl in *Erfahrung und Urteil* (1939).[29]
3. That of the *internal temporal coding* of the geometry of vision, especially through synchronization processes in the visual cortex. The issue of internal temporal coding raises fascinating questions concerning the neural implementation of the inner consciousness of time.[30]

Anyway, we hope to have shown that contrary to Husserl's constant claim, there *does* exist a *geometrical* descriptive eidetics able to assume for perception the constitutive tasks of transcendental phenomenology and to mathematize the correlations between the kinetic noetic syntheses and the noematic morphological *Abschattungen*. Using such a geometrical descriptive eidetics, the naturalization of phenomenology can be reduced to the problem of implementing effective algorithms. For that very reason we are convinced that morphodynamical functionalism is the key to naturalizing phenomenology.

Translated by Christopher Macann