Differential Geometrical Performance and Poiesis

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Mathematical Experience and Mathematical Performance

Conventionally, we construe mathematics as knowledge, and the mathematical sign as representation of knowledge. In this essay I propose an alternative approach, pairing mathematical practice as performance with mathematical writing as technology of performance. This foregrounds mathematics as a poietic rather than a tautological practice, and to interpret this poietic practice I outline a materialist phenomenology. I argue that particular attention to differential geometers' practices of diagramming or sketching shapes, manipulating algebra, estimating analytic functions, or tracing kinetic processes offers us a chance to grasp how mathematical signs function outside speech yet in a thoroughly material way. What seems fruitful is to treat these practices not as recording or encoding mathematical entities, but as generating them. In other words, I treat mathematical writing not as a technology of representation but as a technology of embodied performance. Moreover, by looking at differential geometrical writing as a performance practice, I offer some examples of how finite gestures enact smooth ("infinitely differentiable"), "abstract," "objective," or infinite entities via finite traces. By recognizing that geometers exteriorize and materialize their imagination in common technologies of mathematical writing such as the backs of envelopes or chalk and chalkboard, we can avoid the problem of intersubjective communication. Shared and therefore objective mathematical entities are constituted in the interaction of geometers, their disciplinary logics, and technologies.

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In short, we take a materially mediated, performative approach to mathematical experience. Materially mediated means, simply and profoundly, conducting actions in stuff bearing physical attributes like inertia, resistance, friction, decay, and plasticity, so that our material actions constitute our experience. This indirect characterization of materiality in place of physicality leaves open the possibility of nonphysical materials—such as computational media—that also work like wood, water, or plastic in our phenomenological experience. Avoiding the dualist cut between matter (physics) and thought (spirit), perhaps we can retain a materialism that accommodates nonfinite entities, a key point that honors the working mathematician's conventional handling of many infinite entities—whether these be objects (e.g., an infinite end of a minimal surface), spaces (e.g., a set of real-valued functions that has the structure of an infinite-dimensional topological vector space), or a process (e.g., taking the limit of an indexed set of objects where the index is allowed to go to infinity through a continuous range of values). Throughout this work, mathematical entities will generally include processes as well as objects or spaces.

So, with respect to the classical philosophy of mathematics, perhaps the most nonconventional and useful contribution in this investigation is the consideration of differential geometry not as a synchronic map of propositional knowledge, but rather as a performance practice. I consider how geometers in the wild practice geometry at a relatively fine scale of gesture. Formally, a fundamental difference between my project and logic's project is that, whereas logic treats mathematical *proof* as its object of study, I treat mathematical *performance*. For this purpose, I look for several modes of gestural performative experience, provisionally: graphic, discursive, algebraic, and kinematic, emphasizing the graphic. But in order to proceed we require a more precise qualification of the sense in which to understand performance.

On Performance

Marvin Carlson characterized performance by doubling or double-coding—a differentiation between the subject-as-actor and subject-as-audience—and a composition of the acting self and the rule or script that this actor follows, in a setting framed by social conventions distinguishing it from unmarked, everyday activity.[1] The sorts of mathematical performance I consider are those in which a person is enacting, not a script in a doubled consciousness for a spectator, but rather a process of creation.

1. Marvin Carlson, *Performance, a Critical Introduction* (London: Routledge, 1996).

John Austin's speech-act theory powerfully contributed the notion of linguistic utterances that have the ability to change the world materially. An example would be a priest, or a duly appointed officer of the state, saying "By the power invested in me, I hereby pronounce you husband and wife." Certain statements, uttered by certain people under certain circumstances, have the power to cause social, political, even bodily effects. But what differences or similarities are there between such an illocutionary utterance and "Let T be the product of two 2-spheres $S1$ and $S2$," spoken by a geometer during a workshop?

In his ground-breaking books,[2] and in essays such as "Toward a Semiotics of Mathematics," Brian Rotman describes mathematics as being performed by triples of semiotic agents. Following Charles Sanders Peirce, he proposes a tripartite scheme: a Subject who has access to, and only to, the formal Code of mathematics; a Person who uses the heuristic resources of the discipline; and an Agent who acts as a mechanical proxy of the Subject.[3] I take Rotman's work as one of the inspirations for my phenomenological investigation of geometry, but a key difference between his analysis and mine is that I do not rely on an elaborate semiotic theory of the Reasoning Subject, focusing instead on fine-grained materiality and gesture. In fact, as Andrew Pickering suggests, it is not necessary to collapse human agency either to an epiphenomenon of a deterministic or random material system, or to the action of a heroic, and ultimately solipsistic, creator of knowledge.[4] What seems to be worth investigating

2. Brian Rotman, *Ad Infinitum: The Ghost in Turing's Machine. Taking God out of Mathematics and Putting the Body Back In: An Essay in Corporeal Semiotics* (Stanford: Stanford University Press, 1993); idem, *Mathematics as Sign: Writing, Imagining, Counting* (Stanford: Stanford University Press, 2000).

3. This is much too condensed a description, of course. See Brian Rotman's essay, "Toward a Semiotics of Mathematics," in *Mathematics as Sign* (above, n. 2), pp. 1–43.

4. It is worth quoting Pickering in full at this point: "First, the precise trajectory and endpoint of Hamilton's practice were in no way given in advance. Nothing prior to that practice determined its course. Hamilton had, in the real time of his mathematical work, to fix bridgeheads and fillings and to find out in real time just what resistances would emerge relative to intended conceptual alignments—such resistances again could not be foreseen in advance—and to make whatever accommodations he could find to them, with the success or failure of such accommodations itself only becoming apparent in practice. As in our previous examples, then, conceptual practice has to be seen as *temporally emergent,* as do its products. Likewise it is appropriate to note the *posthumanist* aspect of conceptual practice as exemplified in Hamilton's work. My analysis again entails a decentering of the human subject, though this time toward disciplinary agency rather than the material agency that has been at issue in earlier discussions. Here, once more, it is not the case that Hamilton as a human agent disappears from my analysis. I have not sought to reduce him to an "effect" of disciplinary

more carefully is the intricate dance between the human mathematician's "free" choices and the moves demanded by a "disciplined conceptual practice," whether they be "forced moves," free filling moves, or unplanned, unforeseen conceptual associations.[5]

Another difference between this project and Rotman's is that I accept at face value the phenomenological and ethnographic observation that contemporary differential geometers routinely develop rich intuitions and practices working with infinite and smooth entities. The existence and incorporation of such phenomenological evidence requires, then, that I work with a nonfinitistic conception of mathematics and a technology that respects such a nonfinitistic conception. What I do share with Rotman is an interest in sidestepping appeals to the transcendental in favor of secular practice. But if we grant that infinity is not equal to the transcendental, then there might be more than one way to secularize mathematical practice.

These and other theories' different contexts provide diverse senses of performance—but there is always, explicitly or implicitly, a strong presence of an atomic human subject, of ego. By contrast, I use performance in the fine-grained sense of the ephemeral, momentary, temporally embedded gesture, and most crucially in the not-to-be-repeated aspect of intentional and embodied action. As Rotman put it in his essay on gestural writing:

> The gesturo-haptic is a form of writing that exceeds the textual. . . . [Its] importance, value, and strategic or instrumental interest, is not derived from these meanings [but] in the fact of their taking place, and in the subsequent psycho-social-corporeal effects (of affect, safety, assurance, threat, etc.) that they induce and could only induce as a result of having actually occurred, and having done so in the manner, style, and force (all that constitutes what one might call their gestural prosody) in which they did.[6]

Rotman draws a fine distinction between notating movement and capturing movement: notation is an arbitrary assignment of a sign to a gesture, and if the assignment is deterministic, then we say it is

agency, and I do not think that that can sensibly be done. It is rather that the center of gravity of my account is positioned *between* Hamilton as a classical human agent and the disciplines that carried him along. To be more precise, at the center of my account is the dance of intertwined human and disciplinary agency that traced out the trajectory of Hamilton's practice" (Andrew Pickering, *The Mangle of Practice: Time, Agency, and Science* [Chicago: University of Chicago Press, 1995], pp. 140–141). Emphasis in the original.

5. Ibid., pp. 115–119.

6. Brian Rotman, "Corporeal or Gesturo-haptic Writing," *Configurations* 10 (2002): 434–435.

an encoding; by contrast, capture makes a trace that is physically homologous to the original gesture. Rotman is right to point out the distinction between these different ways of relating to movement—however, both are forms of recording. I would take one step from Rotman's recording of motion to emphasize performance over notating as well as capturing, since the latter two actions both connote recording and playback. Moreover, this does not rely on a theory of the subject, a theoretical simplification that is justified by appealing to a field-theoretic version of a monist ontology.[7] Much of my attention is directed toward such temporally saturated aspects of non-ego-centered performance.

This view of mathematics as a performative art/practice impels us to consider its technologies of mathematical writing.

What's Missing from Phonetic Language? Writing

The basic question before us is, can we think without using words (in a language)? The standard way to avoid tautology is to investigate what is, or what characterizes, thinking. But another way is to enlarge the frame to include at least the union of complexes of physical, social, and symbolic actions taken by a group of geometers in pursuing what they recognize as "doing mathematics." So, when we say "think," let us, at least for the purposes of this present investigation, intend "do" mathematics. Readers familiar with analogous studies of scientific practice will recognize that I consider the production of mathematical knowledge also embedded within the social-matter field. But this is not strictly a social-constructivist project, as will become clear.

L. E. J. Brouwer and Jacques Hadamard both vigorously argued that there were clearly ways to think mathematics without words. Of course any musician or artist could report the same, but their testimonies have traditionally been suspect in scholarly discourse since Plato. Geometers, musicians, and artists cannot be relied upon to report their own experiences in ordinary written language, since written language is ipso facto not their medium of articulation. It is essential to note here that I am not claiming that there is *no* medium of articulation for doing mathematics. Quite the contrary, this essay presents differential geometrical writing as a medium, albeit nonverbal, of mathematical articulation. In fact, Hadamard can be criticized for reducing thought to a psychological mystery—an unmedi-

7. On monism, see David Woodruff Smith's description of Husserl's ontology (below, n. 27). And for an example of a field-theoretic ontology, see Gilles Deleuze and Félix Guattari, *A Thousand Plateaus: Capitalism and Schizophrenia* (Minneapolis: University of Minnesota Press, 1987).

ated process in an inaccessible part of an individual mind. I argue that the fixed algebraic structures of semiotics and linguistics are inadequate for the purposes of describing mathematical articulation in writing and sketching.

By a linguistic approach to knowledge, I mean the stance that experience is primarily carried by written (or writable) language; that the form of articulation of greatest interest is alphabetic writing used to encode speech, or what Roy Harris called the glottic form of written symbols; that speech and the other temporal forms of articulation are of secondary interest, or even to be discounted. The formal version of the linguistic approach also includes the assumption that language is paradigmatically that which can be described as having a complex of a lexicon, morphology, syntax, and grammar, plus some finite rule system for transformations of the discrete representation. Coupled with the logical approach to knowledge, we have a version of the logico-linguistic paradigm, of which software programming languages are among the most highly evolved results. In chapter 4, "Postulates of Linguistics," of *A Thousand Plateaus,* Gilles Deleuze and Félix Guattari observe that the abstract machine of language (i.e., the linguistic model) is both too abstract and not abstract enough. The linguistic model is not abstract enough because it only treats linguistic elements as constants across all languages, and fails to account for nonlinguistic elements, which would require a notion of "variables of expression" into which we could plug either linguistic or nonlinguistic elements; this further level of abstraction would allow us to accommodate "forms of content" as well as of expression.[8] It is the generalized linguisticism, an enlargement of alphabetic logocentrism including phonocentrism, that is inadequate to this project.

For the purposes of my study, I need to rely on a conception of writing that is more expansive than the categories of text typically

8. "If the abstraction is taken further, one necessarily reaches a level where the pseudo-constants of language are superseded by variables of expression internal to enunciation itself; these variables of expression are then no longer separable from the variables of content. . . . *If the external pragmatics of nonlinguistic factors must be taken into consideration, it is because linguistics itself is inseparable from an internal pragmatics involving its own factors*. It is not enough to take into account the signified, or even the referent, because the very notions of signification and reference are bound up with a supposedly autonomous and constant structure. There is no use constructing a semantics, or even recognizing a certain validity to pragmatics, if they are still pretreated by a phonological or syntactical machine. For a true abstract machine pertains to an assemblage in its entirety: it is defined as the diagram of that assemblage. It is not language based but diagrammatic and super-linear" (Deleuze and Guattari, *Thousand Plateaus* [above, n. 7], p. 91; emphasis added).

produced and registered by literary practice. Other notions of writing are available. Roy Harris characterizes the typical concept of writing as telementationalist: as a technology for recording a writer's thoughts in a nonvolatile medium, transporting this nonvolatile recording to a remote reader, and decoding the meaning by the receiver.[9] One formal limit of such a telementationalist model of communication is Claude Shannon's information theory, based on an abstraction of cable communication. Such a theory has severe difficulties when we try to use it to understand complex distributed human activities because it ignores the material media—how signs are made, by hand or keyboard, makes a difference in their signification and value; it is subject to the inadequacies of what Michael Reddy called the conduit metaphor;[10] it ignores nonlocal field effects; it ignores the power of synchronization; and it ignores the integrationalist (Harris's term) or coordinative (Anatol Holt's term) functions of writing. To elaborate, the last four points are related to one another by what I call a field-theoretic notion of human organized activity—that is, a notion that is inspired by a continuous distribution of values over a continuous entity (like a smooth manifold) and does not depend a priori on a discrete structure like a graph.[11] A field-theoretic structure admits more readily concepts such as nonlocal (integral) measures, and sidesteps questions like piecewise arc connectivity and complexity, which are often artifacts of the theoretical structures imposed on a phenomenon such as organized human activity. Anatol Holt's answer to his own question: "What are computers for?" was framed in terms of the coordination of human activity.[12] But the coordination—or better put, the organization—of human activity includes not only schematic or topical relations but

9. Roy Harris, *Signs of Writing* (London/New York: Routledge, 1995).

10. Michael J. Reddy, in "The Conduit Metaphor: A Case of Frame Conflict in Our Language about Language," in *Metaphor and Thought,* ed. Andrew Ortony (Cambridge: Cambridge University Press, 1979), pp. 284–324, describes this metaphor as the conceit that "language transfers human thought and feelings." Of course, a further problem is that a sender or receiver may not be well defined, as in the case of radio. This is one of Derrida's points in "Signature Event Context": Jacques Derrida, "Signature Event Context," in idem, *Margins of Philosophy,* trans. Alan Bass (Chicago: University of Chicago Press, 1982), pp. 307–330, where he dissects the Shannon structure: Sender → message → Receiver, by replacing each of the three terms in turn by a paradigmatic blank.

11. This central notion figures more prominently when we critique the discretization problems associated with all graphical and numerical approximations to smooth geometric entities. It is also explored in work in preparation with Niklas Damiris and Helga Wild.

12. Anatol W. Holt, *Organized Activity and Its Support by Computer* (Dordrecht/Boston: Kluwer, 1997). Also personal communication, 1998.

temporal relations as well, so the degree and quality of synchronicity are most relevant. Graph structures only become more complex with the a posteriori addition of temporal information, but patterns over continuous manifolds can be described using the rich and supple notions of time-varying fields. I will not pursue this further at this point for two reasons: first, I wish to defer introducing more mathematical concepts; and second, the social-scientific problem of determining what observables can be empirically measured to parameterize topologies or field theories, though of key importance, lies outside the scope of this essay. And it is not necessary to solve this measurement problem in order to derive the streamlining benefits of this conceptual approach to mathematical performance.

Computer languages, despite their simpler formal ontology and explicit operational role in animating machines, are no less problematic than human languages with respect to questions of material embodiment and mathematical performance. Indeed, we see the purest consequences of a logico-linguistic conception of human languaging behavior in the design of formal languages for programming computers. The effective senescence of artificial languages like LISP and FORTRAN and their associate programming environments marks the displacement of several theoretically significant programming-language paradigms in many domains of software engineering.[13] One indicator of such a displacement is the trend over the past decade in the classification and retrieval of text from natural-language techniques to statistical techniques that work only on formal (nonsemantic) differences in the text data.[14] Another indicator

13. This phenomenon of language death deserves study to see whether and how it indicates a shift of the epistemic spectrum among software systems engineers and designers.

14. Natural-language techniques would, for example, try to parse and interpret parts of grammar, and to semantically define equivalence classes in the lexicon; such techniques would of course be sensitive to the language and subject of the corpus. "Purely" statistical techniques would treat the texts as strings of ASCII or Unicode characters, and have the advantage of working uniformly across languages and subjects. Naturally, fusions are attempted. See Douglass R. Cutting, David Karger, and Jan Pedersen, "Constant Interaction-Time Scatter/Gather Browsing of Very Large Document Collections," in *Proceedings of the Sixteenth Annual International ACM SIGIR Conference on Research and Development in Information Retrieval* (New York: ACM Press, 1993), pp. 126–135; Julio Gilarranz, Julio Gonzalo, and Felisa Verdejo, "Language-Independent Text Retrieval with the EuroWorldNet Multilingual Semantic Database," in *Second Workshop on Multilinguality in the Software Industry: The AI Contribution, IJCAI-97,* ed. C. D. Spyropoulos (San Francisco: Morgan Kaufmann, 1997), pp. 9–16; Yiming Yang and Jan O. Pedersen, "A Comparative Study on Feature Selection in Text Categorization," in *Proceedings of ICML-97, 14th International Conference on Machine Learning,* ed. Douglas H. Fisher (San Francisco: Morgan Kaufmann, 1997), pp. 412–420.

is the role played by object-oriented programming languages and design. Taligent, for example, the company founded by IBM and Apple in 1992, represented one limit of software architecture based on the object-oriented programming paradigm. And in the narrow context of geometrical computation, one of the most ambitious attempts to incorporate such a computational paradigm was Charles Gunn et al.'s Oorange, which incorporated 3-D graphics, numerical integration, generalized animation on arbitrary real parameters, and an elegant, powerful, object-oriented scripting language. None of these computational tools to "do" mathematics were broadly adopted beyond the doorsteps of their inventors.

We see from these examples that the fixed elements of semiotics and linguistics, logical schemata and rigorously precise symbolic algebraic systems, are inadequate to the purpose of mathematical articulation by writing and sketching. So we leave computational systems predicated on such linguistic schemata, and turn also from recording to poiesis.

A Conversation between Two Differential Geometers in the Wild

In order to provide some concrete examples of differential geometric practice, I present an edited transcript of a conversation between two differential geometers in vivo, talking about constant mean curvature (CMC) surfaces and minimal surfaces, which have been the subject of significant contemporary research activity in the area between geometry and analysis. Speaker A is more familiar with CMC surfaces:

A: Remember last time we know now that $M_{0,3}$ surfaces, constant mean curvature surfaces with three ends, have a planar symmetry . . .

B: Wait can we see how that goes again?

(*A picks up chalk and goes to board. See Fig. 1.*)

A: Okay, so we're looking at complete, finite total curvature surfaces in R3, with constant mean curvature. (*Fig. 2. Both people implicitly notice and know that these ends go out to infinity.*) In general we don't care about the topology inside a compact set (*A circles finger around a region in the middle. See Fig. 3.*), but for this classification, we can assume that the surface is genus 0, without loss of generality. (*A erases the "holes" in the middle. See Fig. 4.*) Now, the balancing formula tells us that the three ends must all lie in the same plane.

B: Wait. You mean that these ends have some sort of axis?

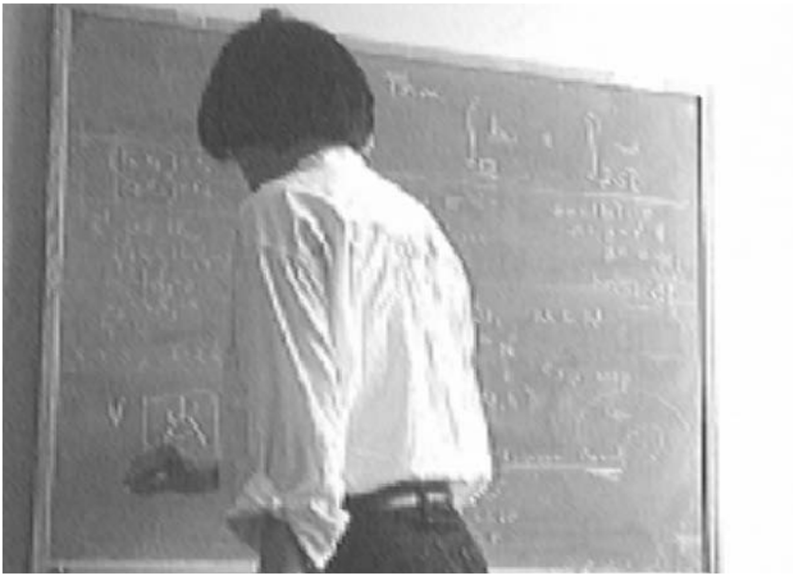

Figure 1. A rudimentary example of a classical geometrical writing and reasoning system.

A: Right, what you do is to integrate the co-normal around a loop around each end and add up the integrals (*see Fig. 5*). Basically it's Stokes's theorem. (*A writes out integral formula expressing the balancing condition. The balancing formula is an analytic relay result proved using purely analytic methods, but later given a pseudo-physical interpretation in terms of flux of fields across membranes*:[15]

$$\sum_{i=1}^{n} n_i \,(2\pi - n_i)\, \bar{a}_i \equiv 0,$$

where n_i *are the asymptotic neck-sizes of the i-th end, and* $\bar{a}_i$ *is the i-th end's axis vector.*)

B: I remember R told me a physical intuition for this. Like if you cap off the ends, you measure the force as a pressure difference across the cap. If the surface is in equilibrium you get the balancing formula.

15. Karsten Grosse-Brauckmann, Robert B. Kusner, and John M. Sullivan, "Triunduloids: Embedded Constant Mean Curvature Surfaces with Three Ends and Genus Zero," *Journal für die reine und angewandte Mathematik* (*Crelle's Journal*) 564 (2003): 35–61.

Figure 2. Three-ended CMC surface, positive genus.

Figure 3. Removing a compact region.

Figure 4. CMC surface simplified "without loss of generality."

Figure 5. Normals along a loop as it travels out to infinity along an end.

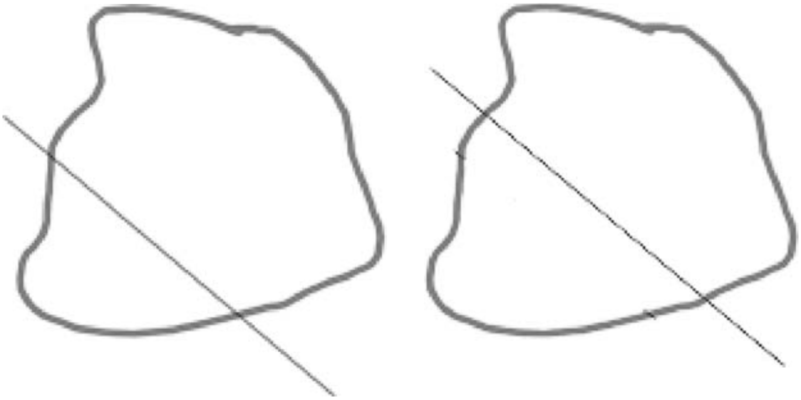

Figure 6. Reflection across a line.

A: Yes that sounds right. You get a weighted sum of the axes vectors. Anyway. The balancing condition tells you that the three axes of the ends have to lie in the same plane. Now you argue that there's a plane of symmetry.

B: The plane containing the axes. (*B waves hand flat over the board.*)

A: Right. Here's where we use the Alexandrov reflection argument. So you know that the only compact genus 0 surface is the sphere. So here's the theorem:

(*A writes the theorem out.*) Theorem (Alexandrov). If *S* is a compact, genus 0 surface of constant mean curvature in R3, then it must be a sphere. So the way that he proved it was to slice the surface with a plane, then reflect across the plane. (*Fig. 6.*) We slide the plane and keep reflecting across it until the first point of contact. Where the surface first touches itself on the inside. (*A taps finger at the point of inner tangency. See Fig. 7.*) And here's where we use the Maximum Principle. So the Maximum Principle says that . . . let's see . . .

(*A and B detour into a discussion of partial differential equations, an analytic theory, about the behavior of solutions to second-order elliptic equations, modeled after* $\Delta u = 0$.)

Remarkably, the two geometers can work anexactly with geometrical entities that cannot be "drawn" because they are infinite in extent, transformations that cannot be graphed because they are elements in spaces that are not merely infinite in cardinality but infinite-dimensional, and entities that are infinite limits. Moreover, these

Figure 7. Reflection across a line with first internal tangency.

geometrical entities' "infinite" aspects play an essential and material role in how they may or may not be manipulated. Yet these two ordinary, fleshy, human mathematicians play their differential-geometrical game with all degrees of evolving intuition and felt logic.[16]

Exteriorization Into Mathematical Machinery, Quasi-Universals, and Objectivity

I take as my starting point the anthropological observation that finite, mortal, fleshy mathematicians collectively share their experience of mathematics over long spans of time. This is not only an ethnographic observation but a phenomenological one as well. We can marvel at what gives mathematics its extraordinary stability across individual experience, but there is no need to account for this stability by an origin of intuition such as occupied Edmund Husserl in his essay "Origin of Geometry." In fact, by appealing to materially mediated experience, I can set aside as well Husserl's Cartesian investigation, with its overtones of idealism. Mathematical thought is exteriorized into its technologies (notation, algebraic methods, computer simulations) as well as its social practices (tacit conventions of how much detail is needed in a publicly accepted proof, accepted procedures for establishing and maintaining truth in a mathematical argument, the coding of visual argument into mathematicians' English, and so forth).

Norton Wise, in a lecture about the projection of British steam technology into eighteenth-century Germany, observed that physical concepts like the conservation of energy arise as universals out of

16. I am indebted to Rafe Mazzeo and Karsten Grosse-Brauckmann for very generously sharing some of their research and process.

the repeated embeddings of a set of material technologies and associated knowledges and expertises into different cultural contexts.[17] As technicians built laboratory apparatuses that supported the physical experiments, they simultaneously built up the conditions for creating symbolic and practical contexts that made sense of quasi-universal notions like heat (or the diffusion model). Bruno Latour, Timothy Lenoir, Andrew Pickering, and many others in Science Studies have followed in detail how complexes of scientific knowledge, physical instruments, laboratory practices, and social relations coconstruct each other in a socio-material setting. It is fruitful to think of mathematics emerging as quasi-universals out of a similar long material circulation. What gives mathematics its peculiarly supple quasi-universality *relative to laboratory science,* however, is the fact that mathematics is simultaneously concept and technology, simultaneously poiesis and techne—both the generation of concepts and the means to articulate and actualize these concepts as material, machinic assemblages.[18] We might say this of language in general, but that degree of generalization loses traction on our investigation of the phenomenology of differential geometrical practice. Mathematical writing can operate in a characteristic way via what mathematicians call, somewhat ironically, mathematical machinery.

Let me give two examples of such machinery: Newton's method for finding roots, and the Maximum Principle as it is used in studying the motion of hypersurfaces under mean curvature flow, or constant mean curvature surfaces as in the example dialogue.

Taking the elementary example first, consider Newton's method for finding the zeroes of a function (see Fig. 8).[19] Start with an initial guess, x_0, for a zero of $f[x]$. Then take the tangent line to $f(x)$ at x_0 and see where it crosses the x-axis at a point that we will call <*a,*0>. $F'(x)$ is the slope of the function $f(x)$ at x_0:

$$f'(x_0) = \frac{f'(x_0) - 0}{x_0 - a}.$$

17. Norton Wise, "Architecture of Steam" (lecture delivered at Stanford University, May 1999).

18. Recall the discussion of materiality in the earlier section on logicism.

19. In fact, in courses for which I prepared Mathematica laboratories, this is how I explicitly presented some derivations and proofs of methods in calculus and differential equations: I showed how a result, such as Newton's method, was constructed, the construction itself constituting a proof of validity of the method; and next I wrote this construction as a procedure in Mathematica, which then could be reexecuted as a function as a component for subsequent work.

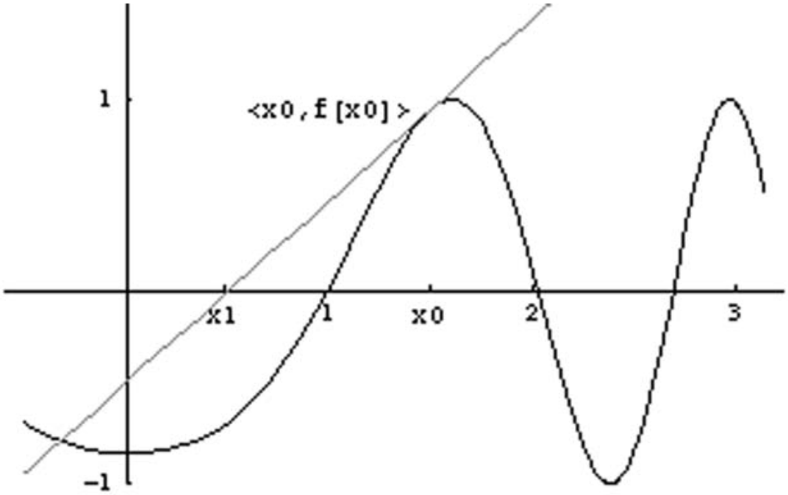

Figure 8. Newton's method for finding roots.

Solve for *a,* where the approximating line intersects the *x*-axis:

$$a = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

Newton's method for approximating the zero of *f* near $x = a$ is the following: Let $x_1 = a$ be the next guess for the zero of $f(x)$. We then derive a recursive formula for successive guesses at the zero of $f(x)$:

$$x_n + 1 = x_n - f[x_n]\, f'[x_n]$$

that we can encapsulate and apply to all sorts of functions, or generalize to other maps on other types of spaces.

Jacques Hadamard argued that this sort of procedure is absolutely typical in mathematics. The encapsulation of a mathematical argument or truth-maintaining procedure into a semiotic form, such as a theorem or formula, that could be easily shared with other mathematicians, he called a Relay-Result.[20] A theoretically coherent set of relay-results with satisfiability conditions, or procedures systematizing actions that can be taken with respect to a set of concepts with provable consequences, is sometimes called "machinery" by mathematicians. This term conjures the composability, inevitability or

20. Jacques Hadamard, *The Mathematician's Mind: The Psychology of Invention in the Mathematical Field* (Princeton: Princeton University Press, 1945), p. 60.

forced logic, and autonomous, hands-free operation of a set of mathematical operations.

Another example, more pertinent to our case studies in the field of differential geometry, is the use of the Maximum Principle in minimal surface theory and the study of flow by mean-curvature. One of the fundamental theorems in the qualitative analysis of solutions of a partial differential equation is the following: Let u be a smooth, real-valued function from a bounded domain W in Rn. Let M be a linear elliptic differential operator defined on a region W by

$$M(u) \equiv xD_i(u)b^i + cux + a^{ij}xD_{ij}(u).$$

Theorem. (Gilbarg-Trudinger 3.1) Suppose $M[u] \geq 0$ on the domain W. Then the maximum value of u is achieved on the boundary.

This theorem and its proof make sense in the domain of real analysis and analysis of partial differential equations. It is established using intuitions and computations from analysis, with no differential geometry. However, this conceptual tool from a substantial machinery—the theory of elliptic partial differential equations—is used over and over in the study of minimal surfaces, but in an essentially visual-kinetic mode. (See Fig. 7.)

In another field and context, differential geometers studying how surfaces evolve under flow by mean curvature first show that two such surfaces satisfy the same elliptic inequality, like $M[u_1] \geq 0$ and $M[u_2] \geq 0$. Then, using the real-analytic[21] fact that M is a linear operator, we have $M(u_2 - u_1) \geq 0$. But by the geometric fact that the surface graphed by u_2 is always below the surface graphed by u_1, we derive the analytic fact that $u_2 - u_1 \leq 0$.

These examples show how a mathematical process like root-finding, or a theorem like the Maximum Principle, can be packaged in such a way that it can be reliably used by other mathematicians—either directly by matching hypotheses and applying the conclusion, or, even more interestingly, in a systematic but analogical way to a new situation. The Maximum Principle is established by methods from real analysis—the existence and regularity theory of partial differential equations, which characteristically uses the notions of calculus, of estimating integral quantities by applying inequalities to sums of Sobolev norms of functions. But its application is strongly geometrical, and even visual. (See Fig. 9.)

21. Real Analysis is a branch of mathematics concerned with the behavior of functions with domain and range in the real number line. It includes the measure theory of sets and the theory of integration. In a precise sense, the space of functions on the real number line is of infinite dimension, and most of these functions are not compatible. (See n. 34 below.)

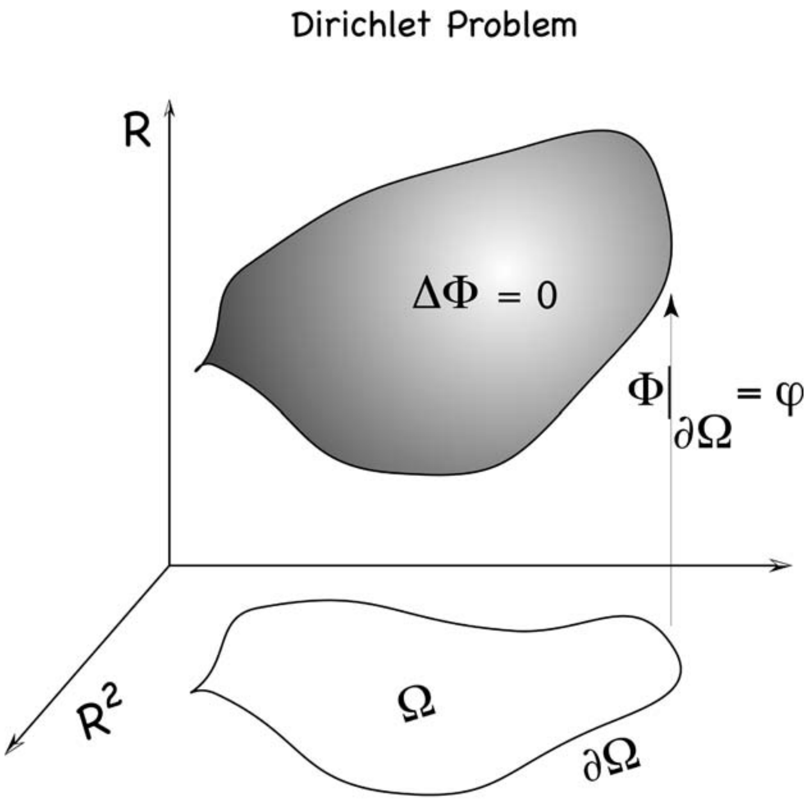

Figure 9. The Dirichlet problem.

In all cases, satisfiability conditions limit and guide the *operational,* not merely descriptive, application of techniques and relay-results, and so here we exceed text, the semiotic, and inscription.

Objectivity and the Technology of Nonglottic Writing

By a technology of writing I mean a conventionalized system of inscription devices, systems of gestural practices and techniques oriented primarily around the production and reception of marks.[22] A technology of writing includes not only the physical instruments but also the practices and historical disciplinary orthography associ-

22. Although my project is focused on visual marks, these marks do not have to be visual. As Harris points out (*Signs of Writing* [above, n. 9]), it is misleading to split an investigation of writing systems a priori along perceptual modalities. Notice that production and reception make no commitments to the notion of transport, so this is not a telementationalist theory.

ated with this technology. As an example of this interrelation between devices and habits, consider TeX. The rise of TeX's prevalence as the standard typesetting system for mathematical texts changed the nature of email-mediated mathematical conversations. We saw earlier some of the inadequacies of telementationalist notions of writing, based on writing as a means of human communication. What are some features of writing that are useful if we want to expand the notion coherently to cover mathematicians' informal sketches and improvisatory algebraic and function-analytic calculations?

For my study of the technologies of mathematical writing, the most fruitful touchstone has been the chalk and blackboard. Keeping in mind how mathematicians have traditionally used chalkboards in their creative and collaborative work, I schematically list a few aspects of writing technologies that appear most salient for this investigation. Writing technologies supporting geometrical exploration involve nonlinguistic as well as linguistic objects; are based upon noncommunicative as well as communicative aspects of language practice; include gestures and other bodily moves coordinated with the written; give material form to abstract entities; are embedded in social practices; enable the gradual and collaborative refinement of notions; and generate "social" objects—objects that are held by and mediate between more than one group of people.[23]

This last point leads us to consider mathematical objects as being humanly constructed, yet objective entities: things that do not belong to any one person, that can be shared and do not fluctuate across cultural positions. In other words, they are aperspectival. This is similar to Paul Ernest's concept of mathematical knowledge as objective beliefs, but with some important differences. In his book, *Social Constructivism as a Philosophy of Mathematics,* Ernest states that

> [t]he social is constituted by individuals together with their shared forms of life. And because of this shared feature, with all of its complexity and human-constituting properties, it cannot be reduced to individuals alone. Thus the objective knowledge which rests in the social is based on shared language use, rules, and understandings, embedded in shared forms of life. . . . [the seeming] necessity . . . of logical and mathematical truth . . . (understood in a fallibilist

23. I am indebted to Helga Wild for discussing these qualities in cogent terms. Many scholars have investigated social objects in related contexts: for example, Latour, borrowing from Michel Serres's quasi-objects (Bruno Latour, *We Have Never Been Modern* [Cambridge, Mass.: Harvard University Press, 1993], p. 51); Susan Leigh Star's boundary objects (Susan Leigh Star and James Griesemer, "Institutional Ecology, 'Translations,' and Boundary Objects: Amateurs and Professionals in Berkeley's Museum of Vertebrate Zoology, 1907–1939," in *The Science Studies Reader,* ed. Mario Biagiolo [London: Routledge, 1999], pp. 505–524).

> sense) rests on linguistic conventions, rules, and accepted social practices, as Wittgenstein proposes.[24]

However, Ernest follows David Bloor in construing mathematics as knowledge, and knowledge as beliefs, whereas I am viewing mathematics as performance practice. So under such a view, I do not have to give an essentialist account of "mathematical truth," which Ernest binds too hastily with logical truth.

Another difference is that Ernest uses the model of "warranting conversations" among mathematicians—characterizing, for example, mathematicians in their work as basically switching between constructing "mathematical knowledge claims" and participating in the "social process of criticism and warranting of others' mathematical knowledge claims."[25] However, a mathematician does much more than invent conjecture: a mathematician performs some algebraic calculation, doodles, balances one function's rate of growth against another, pattern matches against inequalities, and, on a more synthetic level of performance practice, writes out proofs. The model of social conversations (whether intra- or interindividual) does not account for the algebraic, analytic, or logical forces of mathematical argument. And finally, Ernest follows Bloor's theoretical stance: "objectivity is social"—a stance that, as Latour argued, naturalizes the social category as ideologically as realists naturalize quarks.[26] What I am arguing, and what Ernest was trying to arrive at, I believe, is that although mathematical entities are constructed by humans in social contingent contexts, the constructions are actually invariant across subjective, contingent contexts. The social is objective. So how can we resolve social contingency with invariance?

Materialized Phenomenology

This work is critically concerned with the material activities of knowledge-production performed in "real time," based on the assumption that the manner and the material quality of practice strongly affect the kinds of knowledge that get produced. This phenomenological approach regards mathematics as a most human art because it is so free of constraints by any putatively exterior, entirely nonhuman and consequently transcendental Nature. And yet, al-

24. Paul Ernest, *Social Constructivism as a Philosophy of Mathematics,* SUNY Series in Science, Technology, and Society (Albany: State University of New York Press, 1998), p. 146.

25. Ibid., p. 149.

26. David Bloor, *Knowledge and Social Imagery* (London/Boston: Routledge and Kegan Paul, 1976), p. 229, as cited in Ernest, *Social Constructivism,* p. 145; Latour, *We Have Never Been Modern* (above, n. 23).

though its entities are among the most highly crafted of symbolic artifacts, the most artificial, I do not claim that mathematics is immaterial. It is important to understand that Husserl's notion of the essence (*eidos*) of *x* includes the way that *x* is known or experienced; the mode of its conceptual perception is inextricable from its essence. This is key to seeing that Husserl's phenomenology is not an idealist theory but a material one.

This discussion requires an extended notion of materiality, inspired by what David W. Smith calls Husserl's many-aspect monism.[27] What does this mean? In Smith's "unionist" (single world) version of Husserl's ontology, there are material and formal categories. Material categories describe things in terms of particularities, of situatedness and localizability. Formal categories, on the other hand, describe forms that apply across matter, objects, or domains of contingency.[28] Leibniz's *mathesis universalis* notwithstanding, however, it would be a mistake to identify mathematics as a set of formal categories. Indeed, my project is quite the opposite. In Husserl's ontology, material things can be perceived only "from one side," perceived always from a perspective. This perspectivalism is a very useful phenomenological criterion to distinguish "material"[29] from immanent things like experiences and logics, which become known as part of the stream of consciousness, and not from any perspective. Whereas material things are objects of perspectival apperception, immanent things are objects of "eidetic intuition": they are encountered in their essence.

What about mathematical entities? At first, one might think that mathematical entities are "purely formal." But take any mathematical entity—such as curvature as studied in Riemannian geometry, for example. Curvature is encountered by a geometer from many diverse, phenomenologically inequivalent, perspectives. From the perspective of Cartan's method of exterior differential forms, the curva-

27. I use David Woodruff Smith's characterization of Husserl's ontology as a monist, many-aspect continuum rather than idealist ontology. Smith himself later proposed what he called a unionist ontology, in which there is only one world, a significant revision from the singular substance of the monisms of Spinoza and Husserl. See David Woodruff Smith, "Mind and Body," in *The Cambridge Companion to Husserl* (Cambridge: Cambridge University Press, 1995), pp. 323–393; idem, "Intentionality Naturalized?" in *Naturalizing Phenomenology: Issues in Contemporary Phenomenology and Cognitive Science,* ed. Jean Petitot (Stanford: Stanford University Press, 1999), pp. 83–110.

28. I set aside the question of whether the distinction between formal and material categories is substantive or analytic. It is enough for my purposes, following Smith ("Intentionality Naturalized?"), to array mathematical entities across the cross-product of formal with material categories in order to see how they can be both technic and poietic.

29. What Husserl called the "transcendental," somewhat at variance with the modern colloquial sense of the term.

ture can be viewed as a 2-form Ω that can be related to the connection, considered as a 2-form, by the equation

$$d\omega = \Omega + \frac{1}{2}[\omega,\omega].^{30}$$

But from the perspective of, say, a geometric analyst interested in partial differential equations associated with curvature, curvature is viewed as a tensor with components given in terms of the Christoffel symbols Γ^i_{jk} (components of the connection) by the equation

$$R^i_{jkl} = (\Gamma^i_{lj})_{,k} - (\Gamma^i_{kj})_{,l} + \sum_m \Gamma^m_{lj}\,\Gamma^i_{km} - \Gamma^m_{kj}\,\Gamma^i_{lm}.$$

These differences are not only formal but substantive: the effort involved in coming to terms with one or another "perspectival approach" depends very much on individual habits and circumstances; the kinds of actions that one can take with one approach may not be available with the other; the kinds of questions or truths that are compactly and intelligibly stated in component notation may not be expressible in Cartanian notation, and vice versa. Nonetheless, it is also true that a mathematical entity, or a family of entities, once it has been well explored and used by a mathematician, can come to exist in the imagination in a nonperspectival eidetic intuition.

This may be one way to understand what mathematicians mean when they characterize something "well understood" as "trivial." Mathematicians often have the experience that something well understood, typically after a very large amount of perspectival labor, is effortlessly evident and immanent to the imagination, and therefore appears trivial. So, under the richer and more precise ontology made available by a monist—or better, a unionist—phenomenology, we see that mathematical entities can be both material and formal as products of social, and therefore objective, manipulation.

Rotman's suggestion, that mathematical writing provides the shared material medium—technology—that constitutes the intersubjectivity of mathematics, guides us in describing this alinguistic technology as a technology of what Pickering calls disciplinary agency.[31]

30. This is the fundamental structure equation for Riemannian manifolds in Cartan's formulation. See Shoshichi Kobayashi and Katsumi Nomizu, *Foundations of Differential Geometry,* 2 vols. (New York: Wiley, 1996), vol. 1, p. 78.

31. "If mathematical writing is seen as not secondary or posterior to privately engendered intuition, but as constitutive of and folded into the mathematical meaning attached to such a notion, what was private and intrapersonal is revealed as already intersubjective and public" (Brian Rotman, "Thinking Dia-Grams: Mathematics and Writing," in Biagioli, *Science Studies Reader* [above, n. 23], pp. 430–441).

We have approached mathematics as performance under a suitably unmarked sense of everyday skilled practice. This practice consists substantially in making mathematical signs as material traces. If we recognize that the ontology of mathematical entities is the ontology of mathematical signs, it follows that mathematical entities are not transcendental but material. And concretely considering differential geometrical writing as performance, we see particularly clear instances of writing in which substance and form have no categorical separation, and with which the human can write, create, and perform secular, material infinities.

Appendix

Since this essay significantly departs from some philosophical approaches to mathematics, it may be useful to briefly project it against two alternative views of mathematics: L. E. J. Brouwer's intuitionism, and George Lakoff's metaphor theory.

Intuitionism

Brouwer provided one critique of formalist mathematics from the perspective of a working mathematician, arguing against the extensibility by purely logical procedure beyond the scope of phenomenal experience. In his essay "The Unreliability of Logical Principles," he first grounds science as concerned with "repetition in time of sequences in time," implying that these are sequences of the form

a11, a12, a13 . . .
a21, a22, a23 . . .
. . .

whose limits are "ideas."[32] In that passage, he does not explicitly say what his sequences consist of, but his model is that of a sequence of thoughts arising in response to perceptions. According to Brouwer, the reliability of identifications of ideas as limits of such prolongations must derive from perception, and so they are reliable only so long as the mathematical extensions or derivations in the system of entities do not go beyond "the perceptions that make the mathematical system understandable."[33] He seems to think of a mathe-

32. L. E. J. Brouwer, "The Unreliability of Logical Principles," in idem, *Collected Works,* vol. 1, *Philosophy and Foundations of Mathematics,* ed. A. Heyting (New York: American Elsevier, 1975), p. 107.

33. Ibid., p. 107.

matical system as sequences of entities. But it seems that here is a case where Brouwer the philosopher mimicked too closely Brouwer the mathematician, and elevated a notion from real analysis[34]—convergent sequences (sequential compactness)—to a model of mathematical thought. There are many mysteries entangled in this conception of mathematical thought, but let me remark on one of the most striking problems.

The notion of ideas as limits assumes that we think very simply in a linear chain of statements, progressing from one to the next in a unidimensional manner—but there is little neurological or psychological evidence that we in fact do this. Negatively speaking, we can question the presuppositions of a putative cognitive psychology experiment that would observe linearity, because any linearity is more an artifact of the measurement schemas employed than anything in the wet chemistry of our thoughts. In *Gesture and Speech,* André Leroi-Gourhan traced the reduction of writing (hand-drawn sketches and images) over preliterate time to a phonetic transcription of speech, which as a consequence reduced what had been a rich gestural space to a unidimensional semiotic system.[35] Referring to Leroi-Gourhan's philosophized anthropology, Jacques Derrida argued that with the "traditional concept of [unidimensional] time, an entire organization of the world and of language" was bound up with the "linearity of the symbol."[36] But positively, a phenomenological inspection of experience reveals that our consciousness is much more a tissue of shifting moves of assertion, belief, doubt, hope, and so forth. The mathematician Gian-Carlo Rota described this activity in phenomenological terms:

> A subtle form of reductionism may be at work when "evidence" is regarded as an instantaneous process reminiscent of a light bulb being lit. . . . Husserl's description of [the process of acquiring evidence] brings out the complexity of the phenomenon. . . . The gradual discovery of an item beckoning for evidence fragments into a temporality of its own. The imperfection, the lack of certainty, the insecurity of evidence are described by Husserl as features of all evidence whatsoever.[37]

34. On real analysis, see, for example, Halsey L. Royden, *Real Analysis,* 3rd ed. (New York/London: Macmillan, 1988); or Yitzhak Katznelson, *An Introduction to Harmonic Analysis,* 3rd ed. (Cambridge/New York: Cambridge University Press, 2004).

35. André Leroi-Gourhan, *Gesture and Speech* (Cambridge, Mass.: MIT Press, 1993).

36. Jacques Derrida, *Of Grammatology,* trans. Gayatri Chakravorty Spivak (Baltimore: Johns Hopkins University Press, 1976), p. 85.

37. Gian-Carlo Rota, "Kant and Husserl," in *Indiscrete Thoughts,* ed. Fabrizio Palombi (Boston: Birkhauser, 1997), pp. 170–171.

So the view of consciousness that pins it to a unidimensional axis of time, against which we can unambiguously index and order a sequence of concepts, is too simplistic.[38]

But the principal criticism one can make of Brouwer's intuitionism is that he places an unwarranted emphasis on purely cognitive acts. Despite his acknowledgment of the significance of perceptual phenomena grounding logic, he nonetheless worked within a dualist metaphysics of epistemology, a sharp duality between the external world and the internal mind of the mathematical subject. Given such a dualism, it is not surprising that he was led to adopt catechrestically a metaphor from real analysis to account for mathematical thinking, and to amputate much of mathematical (thought) experience in order to remain within reach of finite corporeal experience. I choose to replace the limited scope of Brouwer's "phenomenal experience" in my work with a materially mediated, performative approach to experience.

Mathematics As Metaphor

One of the more popular recent interpretations of mathematical work is that mathematics is primarily metaphorical. George Lakoff and Rafael Núñez are the principal proponents of this theory. Lakoff proposes a useful term: "conceptual blend," which names a very common process where mathematicians adjoin properties to see if interesting new classes of objects are created—but it is predicated on what I think is too limited a reduction of all mathematical relations to metaphorical ones. In *Philosophy in the Flesh,* Lakoff and Mark Johnson characterize metaphor as "conceptual cross-domain mapping."[39] But for them, metaphorical thought is ultimately grounded in "non-metaphorical," "direct physical experience";[40] it is concep-

38. See Husserl's description of "manifold" intensive mental processes, in Edmund Husserl, *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy: First Book, General Introduction to a Pure Phenomenology* [1913], trans. F. Kersten (Dordrech: Kluwer, 1982). Husserl usefully distinguishes between phenomenological and cosmic time (p. 192), and describes "one endless stream of mental processes" (p. 194). The streaming aspect is also useful, but I would not constrain my conception of consciousness, even in the case of mathematical creative reasoning, to a single thread. The study of time would take us very far afield, but see Edmund Husserl, *The Phenomenology of Internal Time-Consciousness* [1928] (Bloomington: Indiana University Press, 1964); William H. Newton-Smith, *The Structure of Time* (London/Boston: Routledge and Kegan Paul, 1980); Paul Horwich, *Asymmetries in Time: Problems in the Philosophy of Science* (Cambridge, Mass.: MIT Press, 1987).

39. George Lakoff and Mark Johnson, *Philosophy in the Flesh: The Embodied Mind and Its Challenge to Western Thought* (New York: Basic Books, 1999), p. v-12.

40. Ibid., p. v-2.

tual; it is used to reason within a categorical way—mapping inferences from one conceptual domain to another conceptual domain. "Each metaphorical idiom comes with a conventional mental image and knowledge about the image. A conventional metaphorical mapping maps the source domain knowledge onto target domain knowledge."[41] For Lakoff, metaphor is a structural homomorphism between two semiotic spaces. But this notion of metaphor seems rather sparse compared to how it is deployed in literary and philosophical discourse.[42]

Such a framework raises a great many problems whose complete discussion would take us too far afield, so I will note only a few. In trying to give their theory a comfortable grounding in physical reality, Lakoff and Johnson state: "All basic sensorimotor concepts are literal. *Cup* (the object you drink from) is literal. *Grasp* (the action of holding) is literal."[43] Since Michel Foucault, however, anthropologists and archaeologists and philosophers of knowledge have become much more careful about naturalizing concepts in such a carefree manner. What constitutes a chair varies so much according to cultural and historical contingencies that it hardly seems like a safe thing to which to anchor a metaphoric superstructure. A more egregious and fundamental problem is Lakoff and Johnson's conflation between "concept of X" and X, where X is a putative physical object. Even if we use their characterization of metaphors as conceptual mappings, it is not at all clear what they mean by their "non-metaphorical concepts" when the putative grounding concepts themselves include culturally conditioned concepts like "cup" and "blond." They tautologically claim that primary metaphors that add sensorimotor inferential structure to nonmetaphorical concepts "are realized in our brains *physically* and are mostly beyond our control. They are a consequence of our brains, our bodies, and the world we inhabit."[44] Finally, this theory assigns metaphor to reasoning, and confines reasoning to the brain (neurology), ignoring the exteriorization of thought into tools, artifacts, and technologies outside the reasoning organism.

What is the problem with the interpretation of mathematical practice as metaphorical thought? There is a logical and even alge-

41. Ibid., p. v-11.

42. See Sheldon Sacks, *On Metaphor* (Chicago: University of Chicago Press, 1979); Andrew Ortony, *Metaphor and Thought,* 2nd ed. (1979; Cambridge: Cambridge University Press, 1993).

43. Lakoff and Johnson, *Philosophy in the Flesh* (above, n. 39), p. 58. Emphasis in original.

44. Ibid., pp. 58–59. Emphasis in original.

braic force to mathematical argument that does not strictly parallel metaphor, and does not reduce to Lakoff and Johnson's metaphorical mapping logic. A simple example of where metaphorical parallelism conflicts with mathematical constraint is the Jordan curve theorem: A simple closed curve in the plane partitions the plane into two connected components. The analogue of this is not true in higher dimensions (the famous counterexample being Alexander's horned sphere). The example is a little too simple because it does not exhibit enough structure to illustrate my argument in full, but I wish to keep the mathematical demands light. Introducing the notion of "disciplinary agency," Andrew Pickering has described an alternative to reductionist physicalism and reductionist social conventionalism, as well as essentially semiotic approaches such as Lakoff and Núñez's metaphor theory.[45] Although his book, *Mangle of Practice,* is primarily concerned with laboratory physics, he devotes one chapter to W. R. Hamilton's construction of quaternions. In this well-known bit of creative mathematical construction, Hamilton worked with and struggled against constraints presented by the algebra over 1, i, j, k defined by these initially unknown relations which he invented:

$$i^2 = j^2 = k^2 = -1;$$
$$ij = k = -ji;$$
$$jk = i = -kj;$$
$$ki = j = -ik.$$

An example from differential geometry is that of the exponential map from a tangent space to a manifold. Analogical metaphor will not suffice to transport the concept of exponential map over to the category where matrix groups are viewed as continuous manifolds. The tangent space of $SL(\mathrm{R}n)$ at the identity is the space of trace-zero matrices. The algebraic structure is not predictable, and does not follow at all automatically from the properties of the real numbers. New proofs and concepts are needed, since it is not clear a priori how to make sense of, for example,

$$\sum_{k=1}^{\infty} \frac{M^k}{k!},$$

where M is a matrix instead of a real number.

45. Pickering, *Mangle of Practice* (above, n. 4).

Lastly, Lakoff's term "basic" in "basic metaphor" is an unfortunate choice, to my mind. Although he claims that this merely means basic as in human-scale, it is no less reductionistic, and no more accurate, than the claim that there are atomic logical entities in mathematics. Moreover, while the notion of human-scale may make sense in a physicalist theory such as Lakoff's metaphor theory, it is not at all clear what "basic" or "human-sized" means when we speak of mathematical entities and operations.[46] Once again, this reduces the force of mathematical interdependencies—the "logic" of mathematics, or what Pickering would call the disciplinary agency of mathematics—solely to human agency, which would land us squarely back in the "paradox" of the intersubjectivity of mathematics: how could the experiences of individual human mathematicians possibly sum to objective mathematics?

46. George Lakoff and Rafael E. Núñez, *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being* (New York: Basic Books, 2000). See also George Lakoff, *Women, Fire, and Dangerous Things: What Categories Reveal about the Mind* (Chicago: University of Chicago Press, 1987).