Motivating Interests

- The "encoding problem" in contemporary mathematical practice.

  Technologies of representation slanted toward recording formal descriptions;

  State of art may support broader spectrum of mathematical working techniques.

- How do people work with "abstract" things via concrete, material marks?

  Grammar driven?

  Object-oriented?

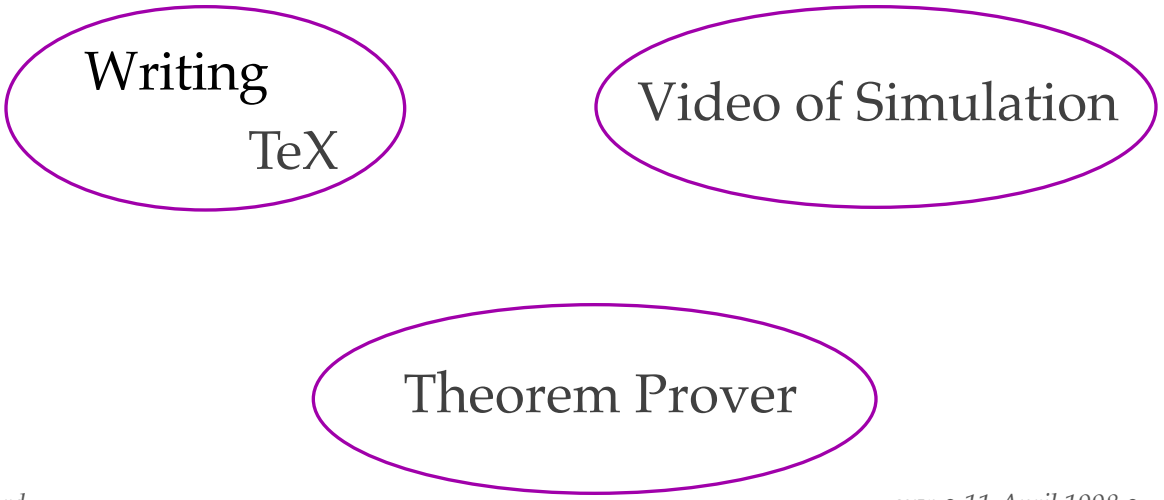
  New models from geometry and topology?

Questions

- What mathematical software is most used by geometers?

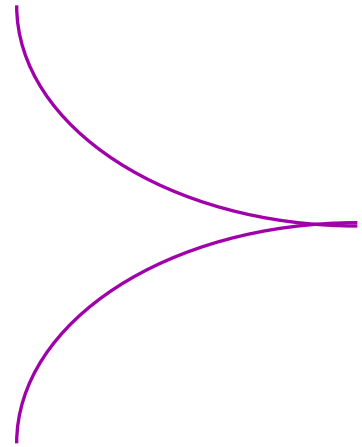
  TeX

- Why?
- Can computers support other practices? How?
- Design space



Geometer's Workbench

writing



blackboard

performing

Motivating Question

- What sort of geometric performance can or cannot be supported computationally in a hybrid writing system that spans

  freehand sketching,

  manipulable diagrams,

  textual description (mathematician's english),

  symbolic algebra,

  numeric simulation?

*Manipulation Modes**

- Discursive

  "Let M *be a compact riemannian manifold of dimension* n*, with metric* g*."*

- Algebraic

  DeclareManifold[M,n,g, {Real, Compact}]

- Analysis **If Ω is a nice domain (Lipschitz), then for $u \in W^{1,p}(\Omega)$:**
- $$\int |u|^{\frac{np}{n-p}} \, dx \leq C \int |Du|^p \, dx$$
- Graphical

**not sense-modality, but phenomenological modality*

Example Scenarios

- Scope
  - Freehand sketch
  - Diagram handle on abstract entitites
  - Steerable numeric computation
  - Steerable symbolic computation
- Depth
  - Geometric evolution problems
  - Noncompact constant mean curvature surfaces

Freehand Sketch

Example 1: Fixed point theorems

- X compact, smooth f: X -> bdyX, then f cannot be id on bdyX.
- f: B^n -> B^n, smooth, then f has fixed point in B.
- f: B^n -> B^n, *continuous*, then f has fixed point.

  (via Weierstrass, approximation of smooth functions by polynomials)

Freehand Sketch

Example 2: GMT Deformation theorem

- Traditionally expensive

  to draw diagrammatic

  component of proof

  =>

  little practice in using

  diagrams to acceptable

  rigor. Social conventions

  on interpreting sketches

  still evolving.

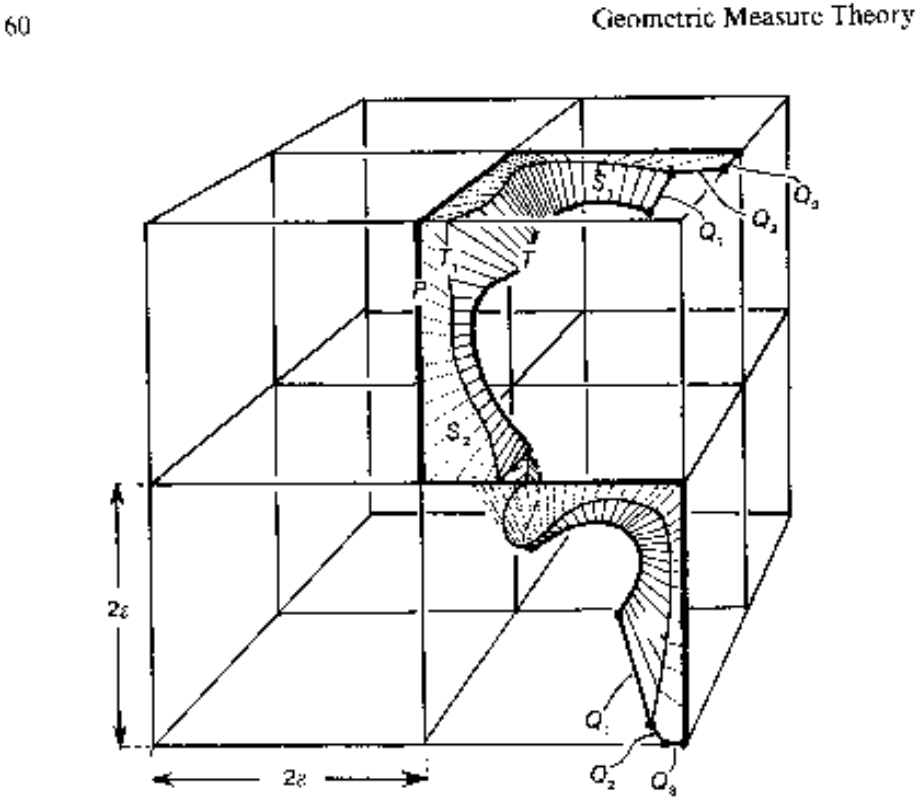
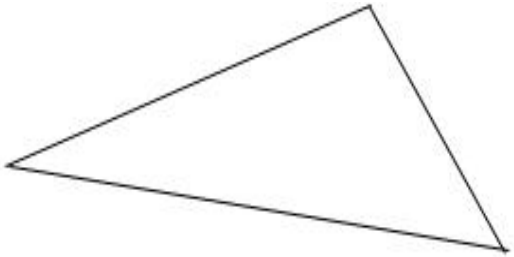
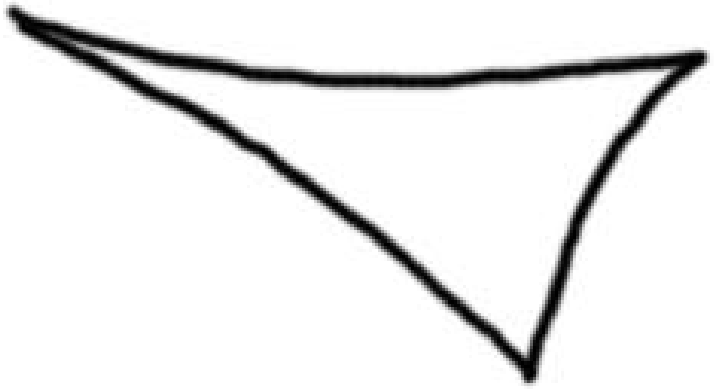


Figure 5.1.1. The Deformation Theorem describes a multi-step process for deforming a given curve T onto a polygon P in the 2ε-grid. During the process surfaces S_1, S_2 are swept out. The endpoints of T trace out curves Q_1, Q_2, Q_3.

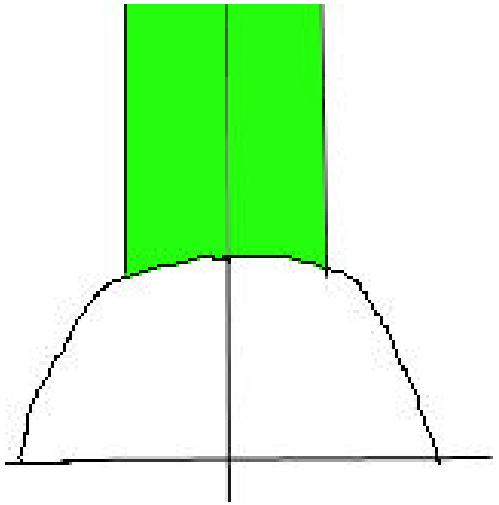
Diagrammatic Manipulator

- Euclidean handle on triangles in non-positively curved space

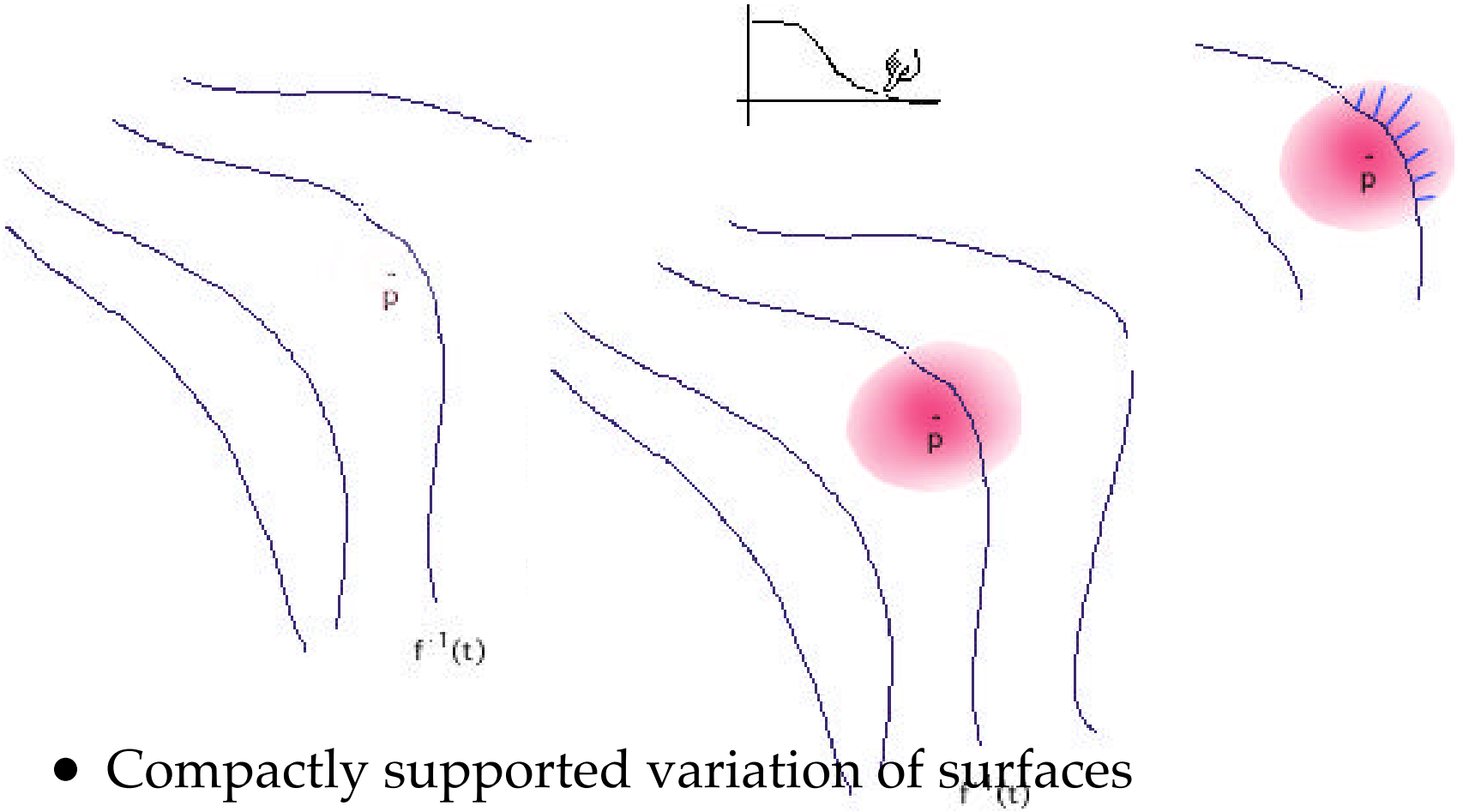




- Handle on spaces: eg. flat tori, generated by (1,0) and (x,y), where ...



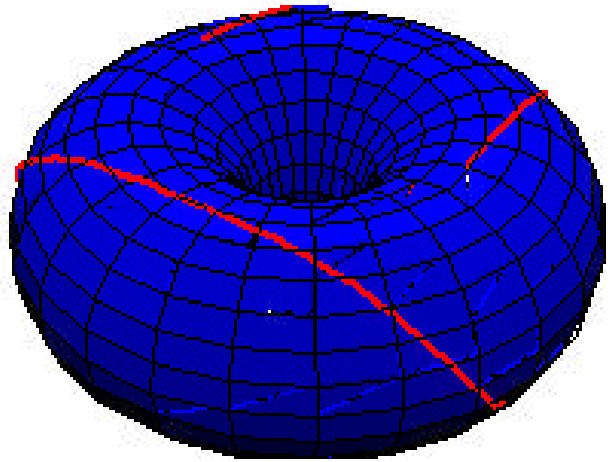
Computational steering

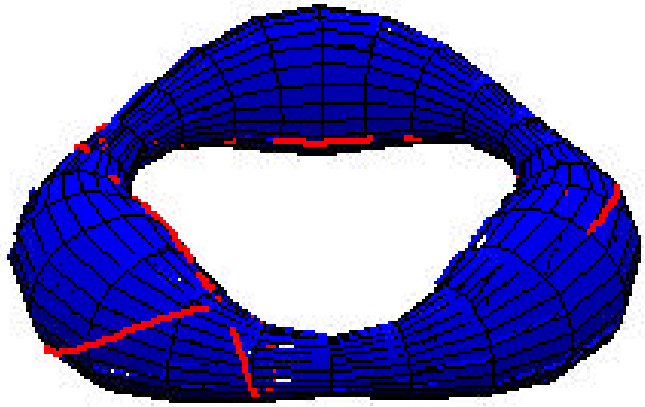


- Compactly supported variation of surfaces
- Spray field

Extended Example: Steerable Computation

- Geodesics on perturbed surfaces
- (Quicktime video)
- Polthier video: discrete geodesics





Design Mantras

Objectives, Principles

- Represent geometric performance (vs. object)
- Geometry ≠ Graphics

  (Recall fixed point theorems)
- Math ≠ Logic

  Evidence-based intuition (vs. predicate logic)

  Structural play (vs. fixed structure)
- Clarity and efficiency via

  Multi-modal representation

  Leave mathematical heuristic to user

Design Mantras

How: Build a Writing/Sketching System

- Start with blackboard

  Allow graffitti sketching in all modes

- Mural augments blackboard

  memory-erase, elastic-zoom, infinite area, user-indexed mark

- Plug engines for in-depth application

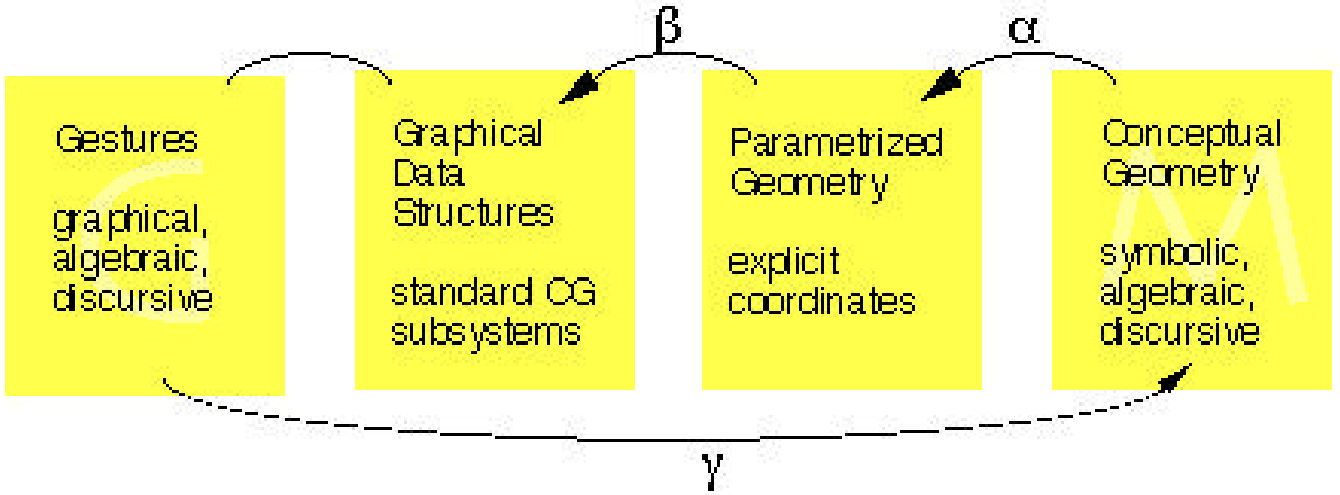
  Support partially-coupled representations (lazy eval)

  Minimize switching cost, allow fine-grain work

- "No Primitive Objects"

Representations

- Mappings are partially faithful



- Prototype in Mathematica, Matlab, OpenGL

Geometer's Workbench

- Discussion