



## References

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### Supplementary

Robert McOwen. Neg, curvat. & conf. def of complte manifolds. 93-98

Heinz-Otto Kreiss theory & numerics pdes

Peter Bjørstad? Domain decomposition for solving biharmonic