

**Is Future Given?
Changes in Our Description of Nature**

Ilya Prigogine's Lecture¹,
after the Honorary Doctorates Ceremony,
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¹Here I give an improved version taking into account the progress reached in the last two years.

According to the classical point of view, nature would be an automaton. However, today we discover instabilities, bifurcations, evolution everywhere. This demands a different formulation of the laws of nature to include probability and time symmetry breaking. We have shown that the difficulties in the classical formulation come from a too narrow point of view concerning the fundamental laws of dynamics (classical or quantum). The classical model has been a model of integrable systems (in the sense of Poincaré). It is this model, which leads to determinism and time reversibility. We have shown that when we leave this model and consider a class of non-integrable systems, the difficulties are overcome. We show that our approach unifies dynamics, thermodynamics and probability theory.

Dear Rector, Dear Deans, Colleagues, Friends,

I

I feel very moved by the kindness shown to me. I don't know if I deserve so many honors. I remember that some years ago a Japanese journalist asked a group of visitors why they are interested in science. My answer was that I feel that science is an important way to understand the nature in which we are living and therefore also our position in this nature. I always felt that there are some difficulties in the descriptions of nature you find currently. I would quote three features. First of all, nature leads to unexpected complexity. This is true on all levels. It is true in the case of the elementary particles; it is true for living systems and, of course, for our brain. The second difficulty is that the classical view does not correspond to the historical time-oriented evolution, which we see everywhere around us. The universe is evolving. That is the main result of modern cosmology with the Big Bang. Everywhere we see narrative stages.

They are events in nature. An event is something, which may or not happen. For example, the position of the moon in one million years is not an event as you can predict it, but the existence of millions of insects we observe is an evidence of what we could call creativity of nature. It is indeed difficult to imagine that the necessary information existed already in some way in the early stages of the universe.

These difficulties have led me to look for a different formulation. This problem is a continuation of the famous controversy between Parmenides and Heraclitus. Parmenides insisted that there is nothing new, that everything was there and will be ever there. This statement is paradoxical because the situation changed before and after he wrote his famous poem. On the other hand, Heraclitus insisted on change. In a sense after Newton's dynamics, it seemed that Parmenides was right, because Newton's theory is a deterministic theory and time is reversible. Therefore nothing new can appear. On the other hand, philosophers were divided. Many great philosophers shared the views of Parmenides. But since the 19th century, since Hegel, Bergson, Heidegger, philosophy took a different point of view. Time is our existential dimension.

I want to show you that the dilemma between Heraclitus and Parmenides can now be put on an exact mathematical framework. As you know, we have inherited from the 19th century two different worldviews. The worldview of dynamics, mechanics and the worldview of thermodynamics. Both views are pessimistic. From the dynamical point of view, everything occurs in a predetermined way. From the thermo-dynamic point of view, everything goes to death, the so-called thermal death. Both points of view are not able to describe the features, which I have mentioned before. Matter was generally considered as a kind of ensemble of dust particles moving in disordered way. Of course, we knew that there are forces. But the forces don't explain the high degree of organization that we find in organisms.

For classical physics and also for quantum physics, there is no privileged direction of time. Future and past play the same role. Since we see an evolutionary universe on all levels of observation. The traditional description is deterministic, even in quantum theory. Indeed, once we know the wave function for one time, we can predict it for arbitrary future or past. This I felt always to be very difficult to accept. I liked the statement by Bergson: time is “invention”.

But the results obtained by classical or quantum mechanics or classical thermodynamics contain certainly a large part of truth. Therefore, the path which I followed over my whole life, was to show that these descriptions are based on a too restricted form of dynamics. We have to introduce a more general starting point. The first step in this direction was an observation, which I made at the beginning of my PhD, in 1945 [1], that non-equilibrium leads to structure. For example, if you consider a box containing two components, say N_2 and O_2 , and you heat it from one side and cool it from the other, you see a difference of concentrations. For example, N_2 may be more concentrated at the hot side. Of course, when you consider the box in thermal equilibrium, the concentrations become uniform. Much later, thanks to the collaboration with Prof. Glansdorff [2], we found that far from equilibrium there appears what we called dissipative structures. These new structures have become quite popular, everywhere one speaks about non-equilibrium structures, self-organization [3–8]. These concepts have been applied to many fields including even social sciences or economic sciences. But I could not stop at this point because thermodynamics is macroscopic physics, so perhaps it is the fact that these systems are large and that we have no exact knowledge of their time evolution and that would give us the illusion of irreversibility. That is the point of view adopted by most people even today. However, my main interest was to show that the difficulty comes from the fact that dynamics, classical or quantum, has to be put on a more general frame.

Let me make here a short excursion on theoretical physics. To describe our nature, we need observables such as space and time. You know that Einstein's great idea was to relate space and time to the properties of matter. But here I do not want to consider relativity, but limit the discussion to classical systems, such as the pendulum, the planetary motion or the motion of particles in a gas. To describe classical systems of this type, we need two kinds of variables: coordinates q and momenta p . In classical theory [9], a dynamical system is described by the so-called Hamiltonian function H . The Hamiltonian is simply the expression of the energy in terms of the observables p and q , $H = H(q, p)$. Once we have the Hamiltonian, we can predict the motion through the so-called canonical equations (the dot means derivative).

$$\dot{p} = \frac{\partial H}{\partial q} \quad \dot{q} = -\frac{\partial H}{\partial p}$$

At the initial time $t = 0$, the observables are the initial positions and momenta q_0, p_0 . As time goes on, they change into $p(t), q(t)$. The observables q, p are called the "canonical variables". Now, a very important point is that there are various choices of canonical variables q and p . This is studied in the basic chapters of classical physics. It is natural to choose the variables q, p , so that the solution of the canonical equations of motion are as simple as possible. It is therefore natural to try to choose them in such a way that we eliminate the potential energy. The Hamiltonian then depends only on the momenta p . We have then $H = H(p)$ and $\dot{p} = 0$. Momenta are constant, therefore the time derivative of the momenta vanishes.

For a long time it was considered that it was always possible to find such "privileged" canonical variables. We could always eliminate the coordinates in the Hamiltonian. But Poincaré, at the end of the 19th century, made a fundamental discovery. He found that this elimination was only possible for a class of dynamical systems, which he called "integrable systems". For

example, in a gas with many particles, this transformation would correspond to going to a representation in which each particle moves independently. When this is possible, the momenta p are called also the action variables J and the coordinates q , the angle variables α . I have to be a little more specific. Consider a system in which the Hamiltonian has two parts

$$H(J, \alpha) = H_0(J) + \lambda V(J, \alpha)$$

We have then one part, H_0 , which depends only on momenta (the action variables), but there is also a perturbation λV depending on both J and α . λ is a parameter measuring the intensity of the perturbation. By definition, for H_0 , we know the action variables. Then for H including λV , we ask if we can construct new action variables, \bar{J} , which would depend analytically on the old ones. This means that the Hamiltonian H can be written as $H(\bar{J})$ with $\bar{J} = J + \lambda J^{(1)} + \lambda^2 J^{(2)} + \dots$

What is the meaning of action variables? They represent *independent* objects, as interactions are eliminated or better to say included in the definition of these objects. This transformation theory has been intensively studied in the 19th and 20th centuries. We can in general introduce new momenta and new coordinates related to p and q by

$$\bar{p} = U^{-1}p, \quad \bar{q} = U^{-1}q$$

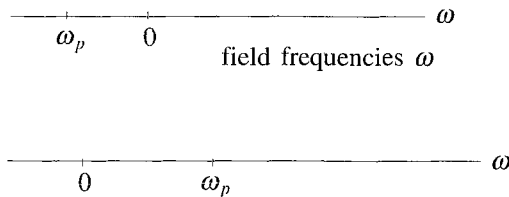
where U is a so-called canonical transformation which preserves the form of the Hamiltonian equations. The analog of U in quantum mechanics is the so-called unitary operator which preserves the form of the Schroedinger equation. U plays an essential role both in classical and quantum mechanics. An important property is the distributivity of U . That means U acting on a product is equal to the product of the transformed entities: $U^{-1}(AB) = (U^{-1}A)(U^{-1}B)$. There are other remarkable properties of unitary transformations, but there is no place here to go further into this [10, 11].

It is remarkable that orthodox quantum mechanics used classical integrable dynamical systems as a model. The basic

difference is that the observables are now no more numbers but *operators*. There are again various representations of the operators related by unitary transformations. Let us only remind that, according to every book on quantum mechanics, in the representation in which q is a number, p is the differentiation operator $i\frac{\partial}{\partial q}$ and we have the commutation relation $qp - pq = \frac{\hbar}{i}$. This is the basis of the Heisenberg's uncertainty relations [10, 11]. For non-integrable systems, the situation, as we shall see now, is quite different.

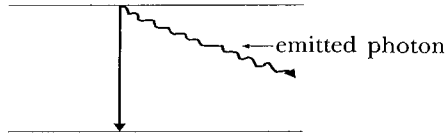
II

After this short introduction to integrable systems, we shall now discuss non-integrable systems. There are of course many classes of non-integrable systems, that is of systems for which we cannot construct a unitary transformation, which eliminates the interactions. We shall consider a specific class of non-integrable systems. That is the class where there exist resonances. What is a resonance? Consider a particle, like a harmonic oscillator, in a field like in electromagnetism. Suppose that the particle frequency is ω_p while the field forms a continuous set of frequencies starting from 0.

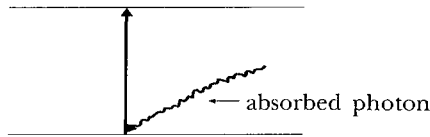


Then there are two situations, either the frequency of the oscillator ω_p is below all the frequencies of the field or the frequency of the oscillator is somewhere in the domain of the frequencies of the field. These are two very different situations. If the frequency of the oscillator is outside the field,

nothing special happens. But if it is inside, we have a so-called excited state and this excited state decays by emitting a photon to a ground state.



This is the well known Einstein and Bohr mechanism for the description of spectral lines. It is generally expressed by saying that the particle is dissolved in the continuum. We have a desexcitation process. There exists of course also an excitation process when the photon falls on the ground state.



The interactions between the field and the oscillators are described by resonances. The fundamental result of Poincaré was to show that such resonances lead to difficulties through the appearance of divergent terms due to small denominators. An example is the term:

$$\frac{1}{\omega_k - \omega_l}$$

ω_k is the frequency of the particle, ω_l is the frequency of the field.

This difficulty was already known to Laplace. How to overcome this difficulty? We have shown that the resonances can be avoided by suitable “analytic continuation”; that means that one has to put small quantities in the denominator to avoid the infinities. Of course, there are some specific

mathematical problems to be overcome here, but they can be studied in the original papers [3, 12–21].

In short, our key idea was to eliminate the Poincaré divergences by extending the idea of unitary transformations. Instead of the formula we have already written for unitary transformation, $\bar{q} = U^{-1}q$, $\bar{p} = U^{-1}p$, we now have:

$$\bar{q} = \Lambda^{-1}q, \quad \bar{p} = \Lambda^{-1}p.$$

The unitary operator U has been replaced by the operator Λ , which is a **star-unitary operator**, but that doesn't matter here. The main point is that we have an extension of canonical transformations. In other words, we have now a new representation of observables and an extension of the dynamical theory. Even in classical theory, it is very important to choose the right representation. For example, if you consider a crystal with vibrating atoms, you can always find a representation in which you have normal coordinates, that means independent motions and then you can define the basic frequencies (normal modes). Similarly here by using the new representation, you can come to expressions of motions, classical or quantum, in which there appear quantities such as transport coefficients, reactions rates, approach to equilibrium. Now the **Λ transformation**, which replaces U , has very interesting new properties. First of all, it is a *non-local* transformation. In other words, classically people were thinking in terms of points, but here we have to speak in terms of ensembles, collections of points. We cannot make a physics of points anymore, we have to make a physics of distributions. This means that we have a statistical description. This also means that we have to give up classical determinism. The second fundamental property of Λ is that we have no more distributivity. More precisely we have $\Lambda^{-1}AB \neq \Lambda^{-1}A \cdot \Lambda^{-1}B$. This opens a whole new domain of classical and quantum physics. We have the appearance of new fluctuations and new uncertainty relations. For example, the Λ operator acting on a product of coordinates is not the product

of the transformed coordinates. There is an uncertainty in position. Let me give an example. In statistical physics, an important role is played by the so-called Langevin equation, where γ is the friction, and noise.

$$\begin{aligned} dp_1(t)/dt &= -\gamma p_1(t) - m\omega_1^2 x_1(t) + B(t) \\ dx_1(t)/dt &= -\gamma x_1(t) + p_1(t)/m + A(t) \end{aligned}$$

These equations describe the damped harmonic oscillator with random momentum. This corresponds, for example, to the motion of a heavy particle in a thermal medium and it is one of the most important results of statistical physics.

Now recently S. Kim and G. Ordonez have shown [87] that using our new transformation Λ , you derive exactly the Langevin equations and therefore also the basic properties studied in statistical mechanics. The Langevin equation has a broken time symmetry. This is not due to approximation but expresses that $x(t)$ and $p(t)$ are Λ transforms. The Langevin equation corresponds to a system in which resonances between the Brownian particle and the thermal medium play an essential role. We have also obtained the *quantum* Langevin equation using the quantum analogues of Λ transforms. Uncertainty relation can now be established for x and p separately. The whole space-time structure is altered. These are fundamental results. Dynamics and probability theory were always considered as separate domains. In other words, statistical theory, noise, kinetic equations were considered as coming from approximations introduced into dynamics, being classical or quantum. What we show now is that these properties, noise and stochasticity are directly derived from a more general formulation of dynamics. These are consequences of non-integrability while integrable systems, which were used as a model for classical and quantum physics, refer in fact only to exceptional ideal cases. We are living in a nature in which the rule is non-integrability. And in non-integrable systems we have quite new properties. The new properties are: First of all, the

appearance of new fluctuations, therefore no more determinism. Secondly, the appearance of a privileged direction of time, that is due to the analytic continuation and third, non-distributivity leading to new uncertainty relations, even in classical physics.

These new properties originate from the fact that we use analytic continuation of the evolution operators. As a result the analytic continuation of a product is not the product of the analytic continuations. When we observe the Langevin equation, the coordinate x and the momentum p have to be understood as non-unitary transforms of the original variables. The new transformed variables are random, leading to stochasticity and probability. In the classical point of view, we may either start from an individual description or with ensembles. Gibbs and Einstein have shown that thermodynamics is based on the theory of ensembles. These ensembles, as we have already mentioned, were obtained from approximations (“coarse graining”). This is no more so, for our class of non-integrable systems. The ensembles point of view is a consequence of the Λ transformation. Λ transforms a phase point into an ensemble. More precisely, the Liouville equation is transformed into a kinetic equation. This, I believe, closes a controversy, which goes back to Boltzmann (1872) [22, 23].

III

Now we want to go to a different aspect. This aspect is related to a different description of elementary processes, unstable particles or quantum transitions. In a sense, it is very nice that these systems are non-integrable. If you could, in the examples of the interaction between the oscillators and the field, apply a unitary transformation, you would not be able to observe the quantum transitions from one level to the others. Electrons, photons are only observable because they interact and participate in irreversible processes. The basic idea of unitary transformation of integrable systems is that you could, in one way or another,

eliminate interactions. But interactions are a fundamental part of nature which we observe and, in non-integrable systems, interactions cannot be eliminated. Think about a gas. In a gas, even if it is at equilibrium, collisions continue to occur and interactions are never eliminated. Collisions give rise to thermal motion. There are limits to reductionism. We have applied our method to a number of problems such as unstable particles or radiation damping (details can be found in the original publications) [19–21, 24–26].

IV

Once we have irreversibility it is clear that we have also some form of the second law of thermodynamics, that means entropy increase.

Boltzmann had the ambition to become the Darwin of physics. He studied the collisions in dilute systems and showed that you can find a function, which plays the role of entropy. This led to a lot of controversies. Poincaré wrote that there was a basic contradiction: on one hand, to use classical mechanics; on the other hand, to come out with entropy which is time oriented. We can now understand what was the reason. Boltzmann tried to apply classical mechanics to non-integrable systems. Gases cannot be integrable systems, because then they would never go to equilibrium. For example, all momenta would be invariants of motion, which would prevent the system to approach equilibrium. So we need non-integrable systems. And once we have non-integrable systems, then Boltzmann's equations are exact consequences of the extended dynamics.

Indeed, we have shown, together with Tomio Petrosky, Gonzalo Ordonez, Evgueni Karpov and others, that we can formulate the second law in terms of dynamical processes. There were always two points of view. The point of view of Boltzmann, stating that the second law is probabilistic and comes ultimately from our ignorance and the point of view of Planck that for the second law, the entropy production is a consequence of

dynamics. Consider the problem of resonances, which I described a little earlier, where we have shown that the decay of the excited state with the emission of the photon is an irreversible process leading to entropy production. This is not astonishing because, in a sense, an excited state contains “more energy” than the ground state. This supplementary energy can then be distributed on all degrees of freedom of the field. We have shown that the inverse process is also possible; that to bring an atom into an excited state, we need a process which brings negative entropy to the atom, which is then used to excite it. In a sense, our whole vision of the universe around us is an example of non-equilibrium systems. We have particles, with mass, and we have photons, without proper mass. Particles with mass should, from the thermodynamical point of view, dissolve into a continuum. Probably the main event in the history of our universe, in the Big Bang is this differentiation. We have massive particles floating in a bath of zero mass objects like the photons.

Conclusions

We come to a different concept of reality. Laplace and Einstein believed that man is a machine within the cosmic machine. Spinoza said that we are all machines but we don't know it. This does not seem very satisfactory. However, to describe our evolutionary universe, we have only taken very preliminary steps. Science and physics are far from being completed, as some theoretical physicists want us to believe. On the contrary, I think that the various concepts, which I have tried to describe in my lecture, show that we are only at the beginning. We don't know what exactly corresponded to the Big Bang, we don't know what determines the families of particles, we don't know how the biological evolution is evolving. May I finish my lecture by some general remarks. Non-equilibrium physics has given us a better understanding of the mechanism of the emergence of events. Events are associated with bifurcations. The future is

not determined. Especially in this time of globalization and the network revolution, behavior at the individual level is the key factor in shaping the evolution of the entire human species. Just as few particles can alter the macroscopic organization in nature to show the appearance of different dissipative structures. The role of individuals is more important than ever. This leads us to believe that some of our conclusions remain valid in human societies.

A famous saying of Einstein is that time is an “illusion”. Einstein was right for integrable systems but the world around us is basically formed by non-integrable systems. Time is our existential dimension. The results described in this paper show that the conflict between Parmenides and Heraclite can be taken out from its metaphysical context and formulated in terms of modern theory of dynamical systems.

Thank you very much.

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